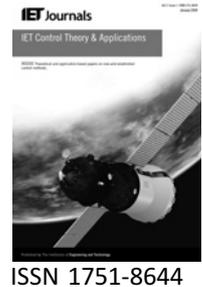


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# Brief Paper

## Stability and stabilisation of discrete-time networked control systems: a new time delay system approach

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**Abstract:** A large number of research works on networked control systems (NCSs) are from the time delay system (TDS) perspective, however, it is noticed that the description of the network-induced delay is too general to represent the practical reality. By recognising this fact, a novel TDS model for NCSs is thus obtained by depicting the network-induced delay more specifically. Based on this model, stability (robust stability) and stabilisation results are obtained using delay-dependent analysis approach, which are less conservative compared with conventional models because of the specific description of the network-induced delay in the new model. A numerical example illustrates the effectiveness of the proposed approach.

### 1 Introduction

With the rapid development of communication technology and embedded devices, the last decade has witnessed the increased use of communication networks in conventional control systems, which forms a novel class of control systems called networked control systems (NCSs). Replacing the perfect data transmission as assumed in conventional control systems by the imperfect communication links, control theory faces a new challenge not met in conventional control systems, that is, the characteristics of the data transmission need also to be investigated carefully in NCSs, since these characteristics greatly affect the system performance of NCSs [1–8].

The most significant characteristic brought by the communication network to the control system in NCSs is certainly the so-called network-induced delay. This delay is caused by transmitting either the sampled data or the control data via the communication network and is inevitable in practice, especially when non-real time communication networks such as the Internet are used

in NCSs. Network-induced delay, together with other communication constraints in NCSs such as data packet dropout, data packet disorder etc., severely degrades the system performance in NCSs, or even destabilises the system at certain conditions. Therefore the study of the effects caused by the network-induced delay in NCSs has formed one of the main themes in the literature on NCSs to date [9–17]. It is noticed that most works in this area are from the perspective of time delay system (TDS) theory since, evidently, NCSs with network-induced delay can be readily modelled as conventional TDSs by simply regarding network-induced delay (probably data packet dropout as well) as a delay parameter to the control system. This modelling approach enables existing results in TDS theory to be applied directly to NCSs and hence significantly simplifies the analysis. Examples of this kind of results can be referred to, for instance [16, 18–23] and the references therein.

In most of the aforementioned literature, network-induced delay is usually assumed to be time varying and yet upper bounded, which is true in practice. However, modelling

NCSs to conventional TDSs with time-varying delays inevitably results in a situation where outdated information may be used at certain conditions even when updated information is already available. This situation occurs when, for example, the network-induced delay at time  $k + 1$  is much larger than that at time  $k$  (please refer to Section 2 for more details). However, the reality is that it is unacceptable to use the outdated information instead of the most updated available in a practical system. This fact implies that the modelling approaches in the aforementioned literature do not represent the reality of practical NCSs well enough. By recognising this fact, a novel model for NCSs is thus proposed in this paper, which appreciates the reality of using the latest information in practical NCSs. This model also leads to less conservative stability and stabilisation results for NCSs, by explicitly taking advantage of the characteristics of the network-induced delay, which cannot be achieved in the methods proposed in the aforementioned literature. The theoretical results are verified by a numerical example which proves the effectiveness of the proposed approach.

The remainder of the paper is organised as follows. In Section 2, we first present the novel model for NCSs, based on which the stability and stabilisation results are then obtained in Section 3. A numerical example is considered to illustrate the effectiveness of the proposed approach in Section 4 and Section 5 concludes the paper.

## 2 Novel TDS model for NCSs

Consider the NCSs set-up illustrated in Fig. 1, where  $\tau_{sc,k}$  and  $\tau_{ca,k}$  are the network-induced delays in the sensor-to-controller and the controller-to-actuator channels, respectively, and the plant is represented by the following discrete-time linear model with the full state information measurable

$$x(k + 1) = Ax(k) + Bu(k) \quad (1)$$

where  $x(k) \in \mathbb{R}^n$  is the state vector,  $u(k) \in \mathbb{R}^m$  is the control input,  $A \in \mathbb{R}^{n \times n}$ ,  $B \in \mathbb{R}^{n \times m}$  are constant system matrices.

For simplicity, in this paper the network-induced delays in the sensor-to-controller and the controller-to-actuator channels are not considered separately but only the round trip delay is of interest, which is denoted by  $\tau_k$  at time  $k$ , that is,  $\tau_k = \tau_{sc,k} + \tau_{ca,k}$ . Using conventional modelling

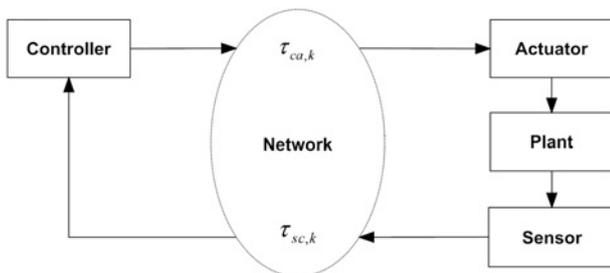


Figure 1 Block diagram of a networked control system

approaches, the control law for the system in (1) is typically obtained as

$$u(k) = Kx(k - \tau_k) \quad (2)$$

where the feedback gain  $K$  is fixed for all the network conditions. In view of the time-varying network conditions, a more reasonable control law is of the following form

$$u(k) = K(\tau_k)x(k - \tau_k) \quad (3)$$

where the feedback gain  $K(\tau_k)$  is designed with respect to different network conditions and thus gives the system designer more freedom to compensate for the communication constraints. It is worth mentioning, however, that although the control law in (3) is more reasonable in practice than that in (2), unfortunately little attention has been paid to (3) in conventional models for NCSs. This is partly because of the fact that in most literature to date, researchers have paid more attention to the analysis and synthesis of the mathematical models for NCSs (TDS models for instance) than to the practical implementation of NCSs. In fact, by taking advantage the specific characteristics of NCSs, the control law in (3) can be implemented in practice by using, for instance, a packet-based control approach [7, 24]. Owing to its advantages, in what follows the control law in (3) is used for further discussion.

In the system models in both (2) and (3), the round trip delay  $\tau_k$  is typically assumed to be time varying and upper bounded. This assumption is generally true in practice as well as necessary in theory. However, this assumption can readily result in a situation where for some specific time  $k'$

$$k' + 1 - \tau_{k'+1} < k' - \tau_{k'} \quad (4)$$

The above inequality means that the control action at time  $k' + 1$  is based on the outdated state information at time  $k' + 1 - \tau_{k'+1}$  instead of the more updated information at time  $k' - \tau_{k'}$  which is already available for the actuator. This control strategy is obviously unacceptable in practice.

By recognising this defect in conventional models for NCSs, we thus have the following reasonable assumption for the network-induced delay in NCSs, denoted by  $\tau_k^*$  to distinguish from  $\tau_k$  in (2) and (3)

$$\tau_{k+1}^* \leq \tau_k^* + 1, \quad \forall k \quad (5)$$

where  $\tau_k^*$  is time varying and upper bounded, that is,  $\tau_k^* \in \Omega \triangleq \{2, \dots, \bar{\tau}\}$ .  $\tau_k^*$  is not less than 2 because of the fact that there is at least one step delay in both the sensor-to-controller and the controller-to-actuator channels, respectively. Notice that the condition in (5) is not naturally held for conventional control approaches to NCSs but can be readily realised in practice by a packet-based control approach where a comparison process is introduced

to avoid the abundant use of the old information. The reader of interest is advised to refer to [7] for the design details of the packet-based control approach.

The control law in (3) can now be rewritten as

$$u(k) = K(\tau_k^*)x(k - \tau_k^*) \quad (6)$$

where  $\tau_k^*$  satisfies (5).

Based on the control law in (6), the closed-loop system for the NCS in (1) with the assumption in (5) can be represented by

$$x(k+1) = Ax(k) + BK(\tau_k^*)x(k - \tau_k^*) \quad (7)$$

where  $\tau_k^*$  satisfies (5) and the feedback gains  $K(\tau_k^*)$  are to be designed. This model is different from conventional models available in the literature in mainly two aspects: (i) the assumption for the network-induced delay in (5) and (ii) the delay-dependent feedback gains  $K(\tau_k^*)$ . Based on this model, stability and stabilisation analysis is then conducted in the following section, which results in less conservative conditions compared with those with conventional models. Another case in the presence of the following time-varying uncertainties will also be considered within this framework

$$x(k+1) = (A + \Delta A(k))x(k) + (B + \Delta B(k))K(\tau_k^*)x(k - \tau_k^*) \quad (8)$$

where the time-varying parameter uncertainties are norm-bounded, that is

$$[\Delta A(k) \ \Delta B(k)] = DE(k)[F_A \ F_B] \quad (9)$$

with  $D$ ,  $F_A$  and  $F_B$  being known constant matrices and

$$E^T(k)E(k) \leq I$$

### 3 Main results

In this section, the stability of the nominal system in (7) is first considered. The result obtained is then extended to the case with time-varying parameter uncertainties in (8). Furthermore, a stabilised controller design method is also obtained in terms of linear matrix inequalities (LMIs).

The following stability theorem for the closed-loop system in (7) can be obtained based on delay-dependent analysis.

*Theorem 1:* Given  $\lambda \geq 1$ . The closed-loop system in (7) is stable if there exist

$$P_i = P_i^T > 0, Q_i = Q_i^T > 0, R_i = R_i^T > 0,$$

$$S_i = \begin{pmatrix} S_i^{11} & S_i^{12} \\ (S_i^{12})^T & S_i^{22} \end{pmatrix} \geq 0, T_i^1, T_i^2$$

with appropriate dimensions such that

1.  $\forall i \in \Omega$

$$\Phi_i = \begin{pmatrix} \Phi_i^{11} & \Phi_i^{12} & (A-I)^T H_i \\ * & \Phi_i^{22} & (BK(i))^T H_i \\ * & * & -H_i \end{pmatrix} < 0 \quad (10)$$

$$\Psi_i = \begin{pmatrix} S_i^{11} & S_i^{12} & T_i^1 \\ * & S_i^{22} & T_i^2 \\ * & * & \frac{1}{\lambda} R_i \end{pmatrix} \geq 0 \quad (11)$$

2.  $\forall i, j \in \Omega$

$$P_i \leq \lambda P_j, Q_i \leq \lambda Q_j, R_i \leq \lambda R_j \quad (12)$$

where

$$\Phi_i^{11} = (\lambda - 1)P_i + Q_i + 2\lambda P_i(A - I) + T_i^1 + (T_i^1)^T + iS_i^{11}$$

$$\Phi_i^{12} = \lambda P_i BK(i) - T_i^1 + (T_i^2)^T + iS_i^{12}$$

$$\Phi_i^{22} = -T_i^2 - (T_i^2)^T + iS_i^{22}$$

$$H_i = \lambda P_i + \bar{\tau} R_i$$

*Proof:* Let

$$z(l) = x(l+1) - x(l) \quad (13)$$

Then

$$x(k) - x(k - \tau_k^*) - \sum_{l=k-\tau_k^*}^{k-1} z(l) = 0 \quad (14)$$

Define the following Lyapunov functional where we suppose at time  $k$ ,  $\tau_k^* = i \in \Omega$

$$V_i(k) = V_i^1(k) + V_i^2(k) + V_i^3(k) \quad (15a)$$

with

$$V_i^1(k) = x^T(k)P_i x(k) \quad (15b)$$

$$V_i^2(k) = \sum_{l=k-\tau_k^*}^{k-1} x^T(l)Q_{\tau_k^*} x(l) \quad (15c)$$

$$V_i^3(k) = \sum_{m=-\tau+1}^0 \sum_{l=k+m-1}^{k-1} z^T(l)R_{\tau_i^*}z(l) \quad (15d)$$

Define  $\Delta V_i(k) = V_{\tau_{k+1}^*}(k+1) - V_i(k)$ . Then along the trajectory of the system in (7), we have

$$\begin{aligned} \Delta V_i^1(k) &= x^T(k+1)P_{\tau_{k+1}^*}x(k+1) - x^T(k)P_i x(k) \\ &\leq (\lambda - 1)x^T(k)P_i x(k) + 2\lambda x^T(k)P_i z(k) \\ &\quad + \lambda z^T(k)P_i z(k) \end{aligned} \quad (16)$$

because of (12)

$$\begin{aligned} \Delta V_i^2(k) &= \left( \sum_{l=k-\tau_{k+1}^*+1}^{k-1} - \sum_{l=k-\tau_k^*}^{k-1} \right) x^T(l)Q_{\tau_i^*}x(l) \\ &\quad + x^T(k)Q_i x(k) \leq x^T(k)Q_i x(k) \end{aligned} \quad (17)$$

because of (5) and

$$\begin{aligned} \Delta V_i^3(k) &= \sum_{m=-\tau+1}^0 \left( \sum_{l=k+m}^k - \sum_{l=k+m-1}^{k-1} \right) z^T(l)R_{\tau_i^*}z(l) \\ &= \bar{\tau}z^T(k)R_i z(k) - \sum_{l=k-\bar{\tau}}^{k-1} z^T(l)R_{\tau_i^*}z(l) \\ &\leq \bar{\tau}z^T(k)R_i z(k) - \sum_{l=k-\tau_k^*}^{k-1} z^T(l)R_{\tau_i^*}z(l) \end{aligned} \quad (18)$$

Notice that

$$z(k) = (A - I)x(k) + BK(i)x(k - \tau_k^*) \quad (19)$$

and

$$R_i \geq \frac{1}{\lambda}R_j, \quad Q_i \geq \frac{1}{\lambda}Q_j, \quad \forall i, j \in \Omega \quad (20)$$

In addition, we have for any  $T_i^1, T_i^2$  with appropriate dimensions

$$\begin{aligned} &2[x^T(k)T_i^1 + x^T(k - \tau_k^*)T_i^2] \times \left[ x(k) - x(k - \tau_k^*) - \sum_{l=k-\tau_k^*}^{k-1} z(l) \right] \\ &= 0 \end{aligned} \quad (21)$$

and for any  $S_i$  with appropriate dimensions

$$i\zeta_1^T(k)S_i \zeta_1(k) - \sum_{l=k-\tau_k^*}^{k-1} \zeta_1^T(k)S_i \zeta_1(k) = 0 \quad (22)$$

where  $\zeta_1(k) = [x^T(k)x^T(k - \tau_k^*)]^T$ .

From (16)–(22) we then obtain

$$\Delta V_i(k) \leq \zeta_1^T(k)\Xi_i \zeta_1(k) - \frac{1}{\lambda} \sum_{l=k-\tau_k^*}^{k-1} \zeta_2^T(k, l)\Psi_i \zeta_2(k, l) \quad (23)$$

where

$$\Xi_i = \begin{pmatrix} \Phi_i^{11} + Y_i^{11} & \Phi_i^{12} + Y_i^{12} \\ * & \Phi_i^{22} + Y_i^{22} \end{pmatrix}$$

$Y_i^{11} = (A - I)^T H_i (A - I)$ ,  $Y_i^{12} = (A - I)^T H_i BK(i)$ ,  $Y_i^{22} = (BK(i))^T H_i BK(i)$ ,  $H_i = \lambda P_i + \bar{\tau}R_i$  and  $\zeta_2(k, l) = [\zeta_1^T(k), z^T(l)]^T$ . It is noticed that the system is stable if  $\Xi_i < 0$  and  $\Psi_i \geq 0$ . Furthermore, noticing that by Schur complement that  $\Xi_i < 0$  is equivalent to  $\Phi_i < 0$ , we then complete the proof.  $\square$

*Remark 1:* In [25], a typical discrete-time system with time-varying state delay was considered, where the Lyapunov functional was constructed with an additional item being (using the notations in this paper)  $V_i^4 = \sum_{m=-\tau+2}^3 \sum_{l=k+1-m}^{k-1} x^T(l)Q_{\tau_i^*}x(l)$ . This item was included mainly to cancel out the first item of the difference of  $\Delta V_i^2(k)$  in (17), since the value of  $(\sum_{l=k-\tau_{k+1}^*+1}^{k-1} - \sum_{l=k-\tau_k^*}^{k-1})x^T(l)Q_{\tau_i^*}x(l)$  cannot be estimated without the assumption in (5) and thus cannot be dropped directly as done in this paper. In [26], the Lyapunov functional used in [25] was further improved by adding another new item to eliminate the negative effect brought by the introduction of  $V_i^4$ . However, without using the item  $V_i^4$  in our Lyapunov functional, a less complex result is obtained in this paper which is also less conservative since no such inequalities are used in the proof. On the other hand, in a recent article [27], a similar delay system was considered in the switched system context, which derived a very similar model to that used in this paper. The aforementioned additional item in the Lyapunov functional  $V_i^4$  was still used, and for the reduction of the coupling under the switched system context, common  $Q$  and  $R$  were used in the Lyapunov functional which obviously led to conservativeness compared with the result in this paper.

Based on Theorem 1, a robust stability theorem can then be obtained for the closed-loop system with time-varying uncertainties in (8).

*Theorem 2:* Given  $\lambda \geq 1$  and the feedback gains  $K(i)$ ,  $i \in \Omega$ . The closed-loop system with time-varying uncertainties in (8) is robust stable if there exist

$$\begin{aligned} &P_i = P_i^T > 0, Q_i = Q_i^T > 0, R_i = R_i^T > 0 \\ &S_i = \begin{pmatrix} S_i^{11} & S_i^{12} \\ (S_i^{12})^T & S_i^{22} \end{pmatrix} \geq 0, T_i^1, T_i^2 \end{aligned}$$

with appropriate dimensions and a scalar  $\gamma > 0$  such that

1.  $\forall i \in \Omega$  (see (24))

$$\Psi_i \geq 0 \tag{25}$$

2.  $\forall i, j \in \Omega$

$$P_i \leq \lambda P_j, Q_i \leq \lambda Q_j, R_i \leq \lambda R_j \tag{26}$$

where  $\Phi_i^{11}, \Phi_i^{12}, \Phi_i^{22}, \Psi_i$  and  $H_i$  are defined in Theorem 1 and  $D, F_A$  and  $F_B$  are defined in (9).

*Proof:* The above theorem can be obtained following a standard analysis for systems with time-varying parameter uncertainties, as done in Theorem 7.3 in [25]. Therefore we omit the technical details for brevity.  $\square$

Based on Theory 1, the following stabilised controller design method can also be obtained in terms of LMIs.

*Theorem 3:* Given  $\lambda \geq 1$ . The system in (7) is stabilisable if there exist

$$L_i = L_i^T > 0, W_i = W_i^T > 0, M_i = M_i^T > 0$$

$$X_i = \begin{pmatrix} X_i^{11} & X_i^{12} \\ (X_i^{12})^T & X_i^{22} \end{pmatrix} \geq 0, Y_i^1, Y_i^2, V_i$$

with appropriate dimensions such that

1.  $\forall i \in \Omega$

$$\Pi_i = \begin{pmatrix} \Pi_i^{11} & \Pi_i^{12} & \lambda L(A-I)^T & \bar{\tau} L(A-I)^T \\ * & \Pi_i^{22} & \lambda (BV_i)^T & \bar{\tau} (BV_i)^T \\ * & * & -\lambda L_i & 0 \\ * & * & * & -\bar{\tau} M_i \end{pmatrix} < 0 \tag{27}$$

$$\Sigma_i = \begin{pmatrix} X_i^{11} & X_i^{12} & Y_i^1 \\ * & X_i^{22} & Y_i^2 \\ * & * & \frac{1}{\lambda} L_i M_i^{-1} L_i \end{pmatrix} \geq 0 \tag{28}$$

2.  $\forall i, j \in \Omega$

$$L_i \leq \lambda L_j, M_i \leq \lambda M_j, W_i \leq \lambda W_j \tag{29}$$

where

$$\Pi_i^{11} = (\lambda - 1)L_i + W_i + 2\lambda(A - I)L_i + Y_i^1 + (Y_i^1)^T + iX_i^{11}$$

$$\begin{aligned} \Pi_i^{12} &= \lambda BV_i - Y_i^1 + (Y_i^2)^T + iX_i^{12} \\ \Pi_i^{22} &= -Y_i^2 - (Y_i^2)^T + iX_i^{22} \end{aligned}$$

Furthermore, the control law is defined in (6) with  $K(i) = V_i L_i^{-1}$ .

*Proof:* The condition in (10) in Theorem 1 can be reformulated as

$$\begin{pmatrix} \Phi_i^{11} & \Phi_i^{12} & \lambda(A - I)^T P_i & \bar{\tau}(A - I)^T R_i \\ * & \Phi_i^{22} & \lambda(BK(i))^T P_i & \bar{\tau}(BK(i))^T R_i \\ * & * & -\lambda P_i & 0 \\ * & * & * & -\bar{\tau} R_i \end{pmatrix} < 0 \tag{30}$$

Pre- and post-multiply (30) and (11) by  $\text{diag}(P_i^{-1}, P_i^{-1}, P_i^{-1}, R_i^{-1})$  and  $\text{diag}(P_i^{-1}, P_i^{-1}, P_i^{-1})$ , respectively, and let  $L_i = P_i^{-1}, M_i = R_i^{-1}, W_i = P_i^{-1} Q_i P_i^{-1}, X_i = \text{diag}(P_i, P_i), S_i = \text{diag}(P_i, P_i), Y_i = P_i^{-1} T_i P_i^{-1}, V_i = K(i) P_i^{-1}$ . We then complete the proof.  $\square$

Theorem 3 provides a way to design a stabilised controller for NCSs in (7). However, the condition in (28) in Theorem 3 is no longer LMI conditions because of the term  $L_i M_i^{-1} L_i$ . To deal with this difficulty, the cone complementarity technique is used in this paper to derive a suboptimal solution for (28) [28, 29], by transforming the problem to a minimisation problem involving LMI conditions.

*Corollary 1:* Given  $\lambda \geq 1$ , define the following non-linear minimisation problem involving LMI conditions for  $i \in \Omega$

$$\mathcal{P}_i: \begin{cases} \text{Minimise } \text{Tr}(Z_i R_i + L_i P_i + M_i Q_i) \\ \text{Subject to (27), (29), } L_i = L_i^T > 0, W_i = W_i^T > 0 \\ M_i = M_i^T > 0, X_i = \begin{pmatrix} X_i^{11} & X_i^{12} \\ (X_i^{12})^T & X_i^{22} \end{pmatrix} \geq 0 \\ \Sigma'_i \geq 0, \Theta_i^1 \geq 0, \Theta_i^2 \geq 0, \Theta_i^3 \geq 0, \Theta_i^4 \geq 0 \end{cases}$$

where

$$\Sigma'_i = \begin{pmatrix} X_i^{11} & X_i^{12} & Y_i^1 \\ * & X_i^{22} & Y_i^2 \\ * & * & \frac{1}{\lambda} Z_i \end{pmatrix}, \Theta_i^1 = \begin{pmatrix} R_i & P_i \\ * & Q_i \end{pmatrix},$$

$$\begin{pmatrix} \Phi_i^{11} + \gamma F_A^T F_A & \Phi_i^{12} + \gamma F_A F_B K(i) & (A - I)H_i & P_i D \\ * & \Phi_i^{22} + \gamma (F_B K(i))^T F_B K(i) & (BK(i))^T H_i & 0 \\ * & * & -H_i & H_i D \\ * & * & * & -\gamma I \end{pmatrix} < 0 \tag{24}$$

$$\Theta_i^2 = \begin{pmatrix} Z_i & I \\ * & R_i \end{pmatrix}, \Theta_i^3 = \begin{pmatrix} L_i & I \\ * & P_i \end{pmatrix}, \Theta_i^4 = \begin{pmatrix} M_i & I \\ * & Q_i \end{pmatrix}$$

If the solution of  $\mathcal{P}_i = 3n, \forall i \in \Omega$ , the system in (7) is then stabilisable with the control law defined in Theorem 3.

For the detailed algorithm based on Corollary 1, the reader is referred to [27, 29].

### 4 Illustrative example

Consider an inverted pendulum system with delayed control input, first discussed in [26]. The discretised model for the system with the sampling period of 30 ms was given by

$$x(k+1) = \begin{pmatrix} 1.0078 & 0.0301 \\ 0.5202 & 1.0078 \end{pmatrix} x(k) + \begin{pmatrix} -0.0001 \\ -0.0053 \end{pmatrix} u(k)$$

and a state feedback gain was obtained in [26] as  $K = [102.9100 \ 80.7916]$ , which is fixed for all network conditions.

In this example, let  $\bar{\tau} = 12$  and thus the network-induced delay in the round trip is time varying within the range [2 12]. In order to generate the delay sequence satisfying (5), a random delay sequence  $\{\tau_k, k \geq 1\}$  is first produced within the range [2 12], which is then modified to obtain  $\{\tau_k^*, k \geq 1\}$  according to (5). This is done in the following ways (i) let  $\tau_1^* = \tau_1$  and (ii) for  $k > 1$ , if  $\tau_{k+1} > \tau_k^* + 1$  then let  $\tau_{k+1}^* = \tau_k^* + 1$ ; let  $\tau_{k+1}^* = \tau_{k+1}$  otherwise. It is worth mentioning that this process of generating the round trip delay sequence  $\{\tau_k^*, k \geq 1\}$  represents the reality in

practical NCSs where only the latest information is used [7]. A typical delay sequence of  $\{\tau_k^*, k \geq 1\}$  is illustrated in Fig. 2, where it is seen that the growth rate of the round trip delay is upper bounded by the dashed lines with their slopes being 1.

Using Corollary 1, the following feedback gains are obtained with respect to different round trip delays

$K(2)$	=	$\begin{pmatrix} 133.1456 & 32.2599 \\ 132.1299 & 31.9383 \end{pmatrix}$
$K(3)$		$\begin{pmatrix} 132.0470 & 31.9097 \\ 132.2548 & 31.9475 \end{pmatrix}$
$K(4)$		$\begin{pmatrix} 131.9716 & 31.8689 \\ 132.3325 & 31.9662 \end{pmatrix}$
$K(5)$		$\begin{pmatrix} 132.1224 & 31.8954 \\ 132.1417 & 31.9062 \end{pmatrix}$
$K(6)$		$\begin{pmatrix} 132.1821 & 31.9203 \\ 132.1446 & 31.9008 \end{pmatrix}$
$K(7)$		$\begin{pmatrix} 131.9822 & 31.8532 \\ \dots & \dots \end{pmatrix}$

The state responses with the control approaches in [26] and in this paper are illustrated in Fig. 3, where it is shown that the system is unstable using the conventional approach in [26] whereas the control approach in this paper can efficiently stabilise the system. This is because of two reasons. Firstly, the stabilised controller design method proposed in this paper takes clear account of the delay constraint in (5) (also illustrated in Fig. 2). Secondly, the use of the time-dependent feedback gains in our model brings more freedom in designing the control law.

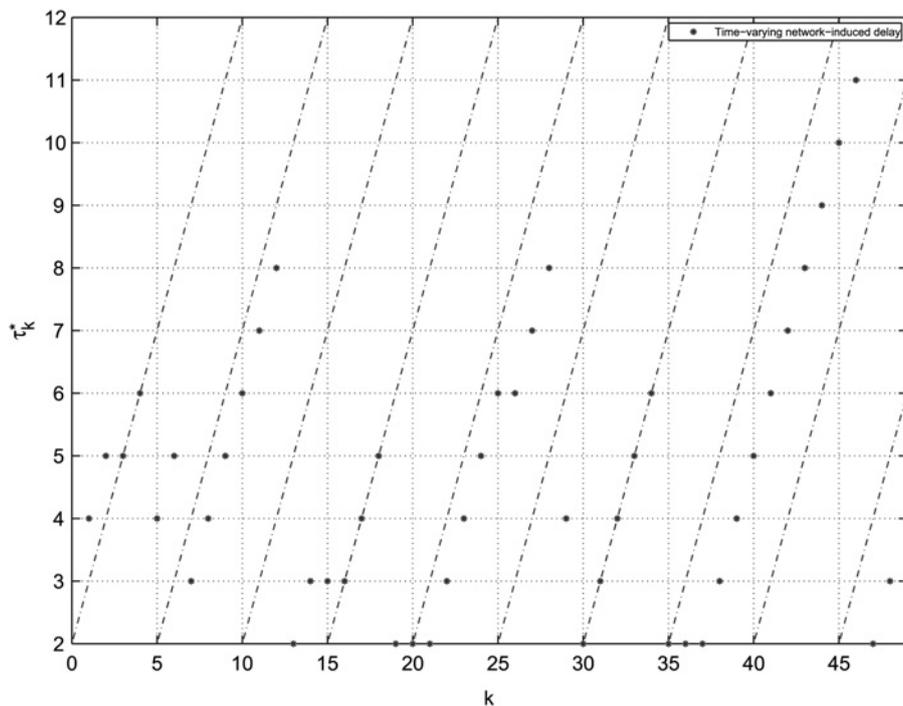
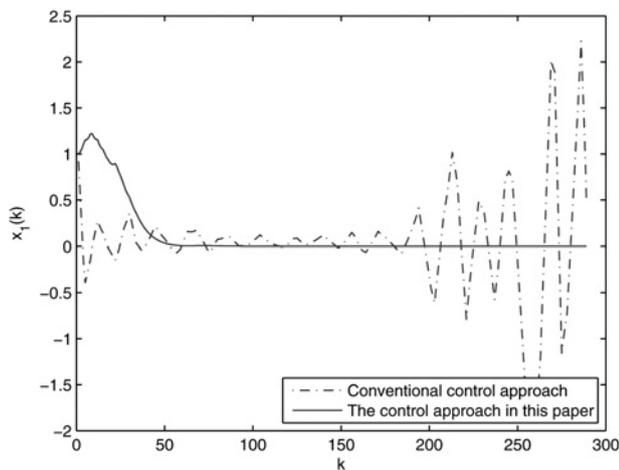


Figure 2 Round trip delay  $\tau_k^*$  which satisfies (5)



**Figure 3** State responses using conventional control approach and the control approach in this paper

## 5 Conclusions

By recognising the reality that only the latest information is used in practical NCSs, a new TDS model for NCSs is proposed. This model takes account of both the specific characteristics of the network-induced delay in practical NCSs and the time-dependent feedback gain scheme. Stability and stabilisation results are obtained based on this model in which less complex Lyapunov functional is used because of the new model. A numerical example illustrates the effectiveness of the proposed approach.

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