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STOCHASTIC STABILIZATION OF PACKET-BASED NETWORKED CONTROL SYSTEMS

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Abstract. *A packet-based control approach is proposed for networked control systems (NCSs). This approach takes advantage of the packet-based transmission of the network in NCSs and as a consequence the control law can be designed with explicit compensation for the network-induced delay, data packet dropout and data packet disorder in both forward and backward channels. Under the Markov chain assumption of the network-induced delay (data packet dropout as well), the sufficient and necessary conditions for the stochastic stability and stabilization of the closed-loop system are obtained. A numerical example illustrates the effectiveness of the proposed approach.*
Keywords: Networked control systems, Communication constraints, Markov chain, Packet-based control

1. Introduction.

Networked Control Systems (NCSs) are such systems where the control loop is closed via some forms of communication network instead of connected directly as assumed in Conventional Control Systems (CCSs) [1]. In NCSs, data is exchanged through a communication network which inevitably introduces communication constraints to the control systems, e.g., network-induced delay, data packet dropout, data packet disorder, data rate constraint, etc. Despite the advantages of the remote and distribute control that NCSs brings, the aforementioned communication constraints in NCSs present a great challenge for conventional control theory [2, 3, 4, 5, 6, 7, 8].

The early work on NCSs has been done mainly from the control theory perspective. Such conventional control theories as time delay system theory [9, 10, 11], stochastic control theory [12, 13, 14, 15], switched system theory [16, 17, 18], have found their applications to NCSs by, typically speaking, modeling the communication network as a negative parameter (mostly a delay parameter) to the system which thus enables a CCS instead of an NCS to be actually considered. These modeling approaches simply ignore the latency of optimizing the system performance by taking advantage of the network characteristics. However, the reality is that the network is not necessarily negative to the system. Although the communication constraints in NCSs normally degrade the system performance, by taking advantage of the data transmission in NCSs, a better system performance can be expected than those using aforementioned conventional control approaches. The preliminary work using this idea, i.e., the so called "co-design" approach, can be seen in, e.g., [19, 20, 21, 22, 23], where the characteristics of the network are analyzed and utilized further.

Most of the work in the co-design area is motivated by the observation of the packet-based data transmission in NCSs [1], which distinguishes NCSs from CCSs. This characteristic can mean that the same amount of network resource is consumed irrespective of whether an NCS sends one single bit or several hundreds bits of data. More specifically, it can be concluded that the same network resource is required to send either a one step control signal or multiple steps of forward control signals within the size limit of the data packets used in NCSs. This observation motivates the study on the so called "packet-based control" for NCSs in this paper, where by designing a special packet-based controller and a corresponding comparison rule at the actuator side, this proposed approach can explicitly compensate for the communication constraints including the network-induced delay, data packet dropout and data packet disorder simultaneously in both forward and backward channels. This merit can not be achieved using conventional control approaches as in, e.g., [12, 24], where the characteristics of the network has not been specially considered and considerable conservativeness is inevitable. Furthermore, with the Markov chain assumption on the round trip delay, the sufficient and necessary condition for the stochastic stability and stabilization of the closed-loop system with the packet-based control approach is obtained, the effectiveness of which is then illustrated by a numerical example.

The remainder of the paper is organized as follows. In Section 2, the problem under consideration is presented, following which the design of the packet-based control approach is then discussed in Section 3. For the derived closed-loop system, the stochastic stability and stabilization results are obtained in Section 4, which is then verified by a numerical example in Section 5. Section 6 concludes the paper.

2. Problem Statement.

The NCS setup considered in this paper is shown in Figure 1, where $\tau_{sc,k}$ and $\tau_{ca,k}$ are the network-induced delays in the backward and forward channels (called "backward channel delay" and "forward channel delay" respectively hereafter) and the plant is linear in discrete-time which can be represented by

$$x(k+1) = Ax(k) + Bu(k) \tag{1}$$

with $x(k) \in \mathbb{R}^n$, $u(k) \in \mathbb{R}^m$, $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times m}$. The full state information is assumed to be available for measurements and the controller to be designed.

It is necessary to point out that the forward channel delay $\tau_{ca,k}$ is not available for the controller when the control action is calculated at time k, since $\tau_{ca,k}$ occurs after the determination of the control action, see Figure 1. For this reason, when applying conventional design techniques such as those in time delay systems to NCSs, the active compensation for the forward channel delay can not be provided. That is, the control law using conventional control approach to NCSs is typically obtained as

$$u(k) = Kx(k - \tau^*_{sc,k} - \tau^*_{ca,k}) \tag{2}$$

where $\tau^*_{sc,k}$ and $\tau^*_{ca,k}$ are the network-induced delays of the control action that is actually applied to the plant at time k and the feedback gain K is fixed for all network conditions. The fact that K is fixed implies that this conventional design technique is conservative in

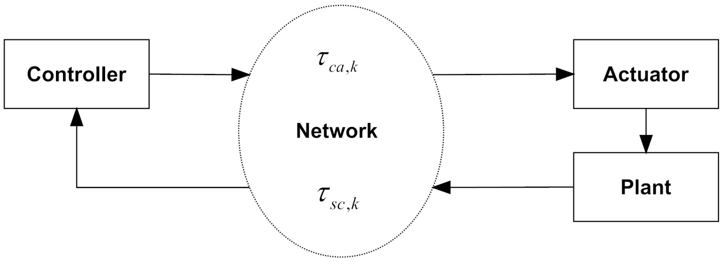

FIGURE 1. The block diagram of a networked control system

the networked control environment, since it loses the capability of actively compensating for the communication constraints while the system is up and running.

By recognizing this deficiency of conventional approaches to NCSs, a packet-based control approach is therefore designed with explicit consideration of the communication constraints in NCSs, which is presented in detail in the next section. The control law based on this approach is obtained as follows when no time-synchronization among the control components is available (Algorithm 3.1),

$$u(k) = K(\tau^*_{sc,k}, \tau^*_{ca,k})x(k - \tau^*_{sc,k} - \tau^*_{ca,k}) \tag{3}$$

when with the time-synchronization (Algorithm 3.2), it is obtained as

$$u(k) = K(\tau^*_k)x(k - \tau^*_k) \tag{4}$$

where $\tau^*_k = \tau^*_{sc,k} + \tau^*_{ca,k}$. It is noted that using the control laws in (3) and (4), the feedback gains can be designed with explicit consideration of the communication constraints, thus enabling us to actively compensate for the communication constraints in NCSs by applying different feedback gains for different network conditions, as is done in Section 4. In the following remark, we notice that other researchers have also attempted to achieve such an advantage which however is not realizable in practice since no supportive design method has been given.

Remark 2.1. *In* [12]*, the authors noticed the unavailability of the forward channel delay and a controller was designed with the following form*

$$u(k) = K(\tau_{sc,k}, \tau_{ca,k-1})x(k - \tau_{sc,k} - \tau_{ca,k}) \tag{5}$$

where the forward channel delay of the last step $\tau_{ca,k-1}$ was used instead. However, actually even $\tau_{ca,k-1}$ is generally unavailable for the controller in practice since in the case of a random forward channel delay, $\tau_{ca,k-1}$ can not be known to the controller until the controller receives information of $\tau_{ca,k-1}$ from the actuator. Therefore, it is seen that $\tau_{ca,k-1}$ can not be available for the controller earlier than time $k - 1 + \tau_{ca,k-1}$ even if an additional delay-free channel exists to send the information of $\tau_{ca,k-1}$ from the actuator to the controller. As a result, the above model in (5) is inappropriate in practice unless a special control structure is designed for the networked control environment as in this paper.

The comparison of the control laws between the packet-based control approach in (3) and (4) and the conventional ones in (2) and (5) (which is not realizable in practice), reveals the superiority of the approach proposed in this paper. To implement the control laws in (3) and (4), the design of the packet-based control approach is then presented in the next section in detail.

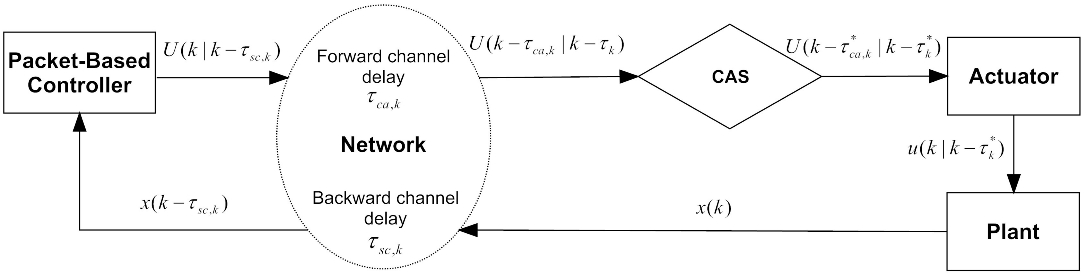

FIGURE 2. Packet-based control for networked control systems

3. **Packet-based Control.** For the design of the packet-based control approach for NCSs, the following assumptions are required.

Assumption 3.1. *The controller and the actuator (plant) are time-synchronized and the data packets sent from both the sensor and the controller are time-stamped.*

Assumption 3.2. *The sum of the maximum forward (backward) channel delay and the maximum number of consecutive data packet dropout (disorder as well) is upper bounded by $\bar{\tau}_{ca}$ ($\bar{\tau}_{sc}$ accordingly) and*

$$\bar{\tau}_{ca} \leq \frac{B_p}{B_c} - 1 \tag{6}$$

where B_p is the size of the effective load of the data packet and B_c is the bits required to encode a single step control signal.

Remark 3.1. *Time-synchronization is required for the implementation of the control law in (3), which can be relaxed for the control law in (4), see Remark 3.4. The practical realization of time-synchronization can be approximately achieved by using the approach in, e.g.,* [25]*, which will not be discussed in detail in this paper. With time-synchronization among the control components and the time stamps used, the network-induced delay that each data packet experiences can then be known by the controller and the actuator upon its arrival.*

Remark 3.2. *In Assumption 3.2, the upper bound of the delay and dropout is only meant for those received successfully; A dropped data packet is not treated as an infinite delay. In light of the UDP (User Datagram Protocol) that is widely used in NCSs, this upper bound assumption is thus reasonable in practice as well as necessary in theory. Furthermore, the constraint in (6) is easy to be satisfied, e.g., $B_p = 368$ bit for Ethernet IEEE 802.3 frame which is often used* [26]*, while an 8-bit data (i.e., $B_c = 8$ bit) can encode $2^8 = 256$ different control actions which is ample for most control implementations; In this case, 45 steps of forward channel delay is allowed by (6) which can actually meet the requirements of most practical control systems.*

The block diagram of the packet-based control structure is illustrated in Figure 2. It is distinct from the conventional control structure in two respects: the specially designed packet-based controller and the corresponding Control Action Selector (CAS) at the actuator side.

In order to implement the control laws in (3) and (4), we take advantage of the packet-based transmission of the network to design a packet-based controller instead of trying to obtain directly the current forward channel delay as this is actually impossible in practice. As for the control law in (3), the packet-based controller determines a sequence of forward control actions as follows and sends them together in one data packet to the actuator,

$$U_1(k|k-\tau_{sc,k}) = [u(k|k-\tau_{sc,k}) \ \dots \ u(k+\bar{\tau}_{ca}|k-\tau_{sc,k})]^T \tag{7}$$

where $u(k+i|k-\tau_{sc,k})$, $i=0,1,\dots,\tau_{ca,k}$ are the forward control action predictions based on information up to time $k-\tau_{sc,k}$.

When a data packet arrives at the actuator, the designed CAS compares its time stamp with the one already in CAS and only the one with the latest time stamp is saved. Denote the forward control sequence already in CAS and the one just arrived by $U_1(k_1-\tau_{ca,k_1}|k_1-\tau_{k_1})$ and $U_1(k_2-\tau_{ca,k_2}|k_2-\tau_{k_2})$ respectively, then the chosen sequence is determined by the following comparison rule,

$$U_1(k-\tau^*_{ca,k}|k-\tau^*_k)=\begin{cases}U_1(k_2-\tau_{ca,k_2}|k_2-\tau_{k_2}), & \text{if } k_1-\tau_{k_1}<k_2-\tau_{k_2};\\ U_1(k_1-\tau_{ca,k_1}|k_1-\tau_{k_1}), & \text{otherwise.}\end{cases} \tag{8}$$

The comparison process is introduced because different data packets may experience different delays thus producing a situation where a packet sent earlier may arrive at the actuator later, that is, data packet disorder. After the comparison process, only the latest available information is used.

CAS also determines the appropriate control action from the forward control sequence $U_1(k-\tau^*_{ca,k}|k-\tau^*_k)$ at each time instant as follows:

$$u(k)=u\left(k|k-\tau^*_{sc,k}-\tau^*_{ca,k}\right) \tag{9}$$

It is necessary to point out that the appropriate control action determined by (9) is always available provided Assumption 3.2 holds and (9) is equivalent to the control law in (3) if state feedback is used, i.e.,

$$u(k)=u\left(k|k-\tau^*_{sc,k}-\tau^*_{ca,k}\right)=K\left(\tau^*_{sc,k},\tau^*_{ca,k}\right)x\left(k-\tau^*_{sc,k}-\tau^*_{ca,k}\right) \tag{10}$$

The packet-based control algorithm with the control law in (3) can now be summarized as follows based on Assumptions 3.1 and 3.2.

Algorithm 3.1. ***Packet-based control with the control law in (3)***

S1. *At time k, if the packet-based controller receives the delayed state data $x(k-\tau_{sc,k})$, then, it*

S1a. *Reads current backward channel delay $\tau_{sc,k}$;*

S1b. *Calculates the forward control sequence as in (7);*

S1c. *Packs $U_1(k|k-\tau_{sc,k})$ and sends it to the actuator in one data packet with time stamps k and $\tau_{sc,k}$.*

If no data packet is received at time k, then let $k=k+1$ and wait for the next time instant.

S2. *CAS updates its forward control sequence by (8) once a data packet arrives;*

S3. *The control action in (10) is picked out from CAS and applied to the plant.*

In practice, it is often the case that we do not need to identify separately the forward and backward channel delays since it is normally the round trip delay that affects the system performance. In such a case, the simpler control law in (4) instead of that in (3) is applied, for which the following assumption is required instead of Assumption 3.2.

Assumption 3.3. *The sum of the maximum network-induced delay and the maximum number of continuous data packet dropout in the round trip is upper bounded by $\bar{\tau}$ and*

$$\bar{\tau}\le\frac{B_p}{B_c}-1 \tag{11}$$

With the above assumption, the packet-based controller is modified as follows:

$$U_2(k|k-\tau_{sc,k})=[u(k-\tau_{sc,k}|k-\tau_{sc,k})\ \dots\ u(k-\tau_{sc,k}+\bar{\tau}|k-\tau_{sc,k})]^T \tag{12}$$

It is noticed that in such a case the backward channel delay $\tau_{sc,k}$ is not required for the controller, since the controller simply produces $(\bar{\tau}+1)$ step forward control actions whenever a data packet containing sensing data arrives. This relaxation implies that the time-synchronization between the controller and the actuator (plant) is not required and thus Assumption 3.1 can then be modified as follows.

Assumption 3.4. *The data packets sent from the sensor are time-stamped.*

The comparison rule in (8) and the determination of the actual control action in (10) remain unchanged since both of them are based on the round trip delay τ_k and in this case the control law with state feedback is obtained as follows, as presented in (4),

$$u(k) = u(k|k - \tau_k^*) = K(\tau_k^*)x(k - \tau_k^*) \tag{13}$$

The packet-based control algorithm with the control law in (4) can now be summarized as follows based on Assumptions 3.3 and 3.4.

Algorithm 3.2. ***Packet-based control with the control law in (4)***

S1. *At time k, if the packet-based controller receives the delayed state data $x(k - \tau_{sc,k})$, then,*

S1a. *Calculates the forward control sequence as in (12);*

S1b. *Packs $U_2(k|k - \tau_{sc,k})$ and sends it to the actuator in one data packet.*

If no data packet is received at time k, then let $k = k + 1$ and wait for the next time instant.

S2 and S3. *remain the same as in Algorithm 3.1.*

Remark 3.3. *From the design procedure of the packet-based control approach, it is seen that the implementation of this approach requires only: 1) a modified controller to produce the sequences of the forward control signals in (7) or (12) and 2) the so designed CAS at the actuator side to compensate for the communication constraints. In practice, the latter could be a separate control component added to the system, and the packet-based controller can be designed using any appropriate methods that can give rise to a good system performance. Therefore, this approach can be readily implemented in practice. Furthermore, the fact that conventional control design methods can still be fitted in the packet-based control framework also makes the proposed approach a universal solution to NCSs.*

4. Stochastic Modeling and Stabilization.

It is noticed that the control law in (3) equals that in (4) if $K(\tau_k^*) = K(\tau_{sc,k}^*, \tau_{ca,k}^*)$ which is generally true in practice. Thus, for simplicity only the closed-loop system with the control law in (4) (i.e., Algorithm 3.2) is analyzed in this paper.

Let $X(k) = \left[x^T(k)\ x^T(k-1)\ \cdots\ x^T(k-\bar{\tau})\right]^T$, then the closed-loop system with the control law in (4) can be written as

$$X(k+1) = \Xi(\tau_k^*)X(k) \tag{14}$$

where $\Xi(\tau_k^*) = \begin{pmatrix} A & \cdots & BK(\tau_k^*) & \cdots & \cdots \\ I_n & & & & 0 \\ & I_n & & & 0 \\ & & \ddots & & \vdots \\ & & & I_n & 0 \end{pmatrix}$ and I_n is the identity matrix with rank n.

4.1. The stochastic model of the packet-based control approach for NCSs.

In NCSs, it is reasonable to model the round trip delay $\{\tau_k; k = 0, 1, \ldots\}$ as a homogeneous ergodic Markov chain [12]. Here in order to take explicit account of the data packet dropout, Markov chain $\{\tau_k; k = 0, 1, \ldots\}$ is assumed to take values from

$\mathcal{M} = \{0, 1, 2, \ldots, \bar{\tau}, \infty\}$ where $\tau_k = 0$ means no delay in round trip while $\tau_k = \infty$ implies a data packet dropout in either the backward or the forward channel. Let the transition probability matrix of $\{\tau_k;\ k = 0, 1, \ldots\}$ be denoted by $\Lambda = [\lambda_{ij}]$ where

$$\lambda_{ij} = P\{\tau_{k+1} = j | \tau_k = i\}, \quad i, j \in \mathcal{M}$$

$P\{\tau_{k+1} = j | \tau_k = i\}$ is the probability of τ_k jumping from state i to j, $\lambda_{ij} \geq 0$ and

$$\sum_{j \in \mathcal{M}} \lambda_{ij} = 1, \quad \forall i, j \in \mathcal{M}$$

The initial distribution of $\{\tau_k;\ k = 0, 1, \ldots\}$ is defined by

$$P\{\tau_0 = i\} = p_i, \quad i \in \mathcal{M}$$

According to the comparison rule in (8), the round trip delay of the control actions that are actually applied to the plant can be determined by the following equation.

$$\tau^*_{k+1} = \begin{cases} \tau^*_k + 1, & \text{if } \tau_{k+1} > \tau^*_k; \\ \tau^*_k - r, & \text{if } \tau^*_k - r = \tau_{k+1} \leq \tau^*_k. \end{cases} \tag{15}$$

Remark 4.1. *The data packet dropout is explicitly considered by including the state $\tau_k = \infty$ into the state space Λ; The data packet disorder is also considered by (15): In our stochastic model the network-induced delay, data packet dropout and data packet disorder are all considered simultaneously. To the best knowledge of the authors, there is no analogous analysis available in the literature to date.*

Lemma 4.1. $\{\tau^*_k;\ k = 0, 1, \ldots\}$ *is a non-homogeneous Markov chain with state space $\mathcal{M}^* = \{0, 1, 2, \ldots, \bar{\tau}\}$ whose transition probability matrix $\Lambda^*(k) = [\lambda^*_{ij}(k)]$ is defined by*

$$\lambda^*_{ij}(k) = \begin{cases} \dfrac{\sum\limits_{l_1 \in \mathcal{M}, l_1 \geq i} \pi_{l_1}(k) \lambda_{l_1 j}}{\sum\limits_{l_1 \in \mathcal{M}, l_1 \geq i} \pi_{l_1}(k)}, & j \leq i; \\ \dfrac{\sum\limits_{l_1 \in \mathcal{M}, l_1 \geq i} \sum\limits_{l_2 \in \mathcal{M}, l_2 > i} \pi_{l_1}(k) \lambda_{l_1 l_2}}{\sum\limits_{l_1 \in \mathcal{M}, l_1 \geq i} \pi_{l_1}(k)}, & j = i + 1; \\ 0, & \textit{otherwise}. \end{cases} \tag{16}$$

where $\pi_j(k) = \sum\limits_{i \in \mathcal{M}} p_i \lambda^{(k)}_{ij}$ and $\lambda^{(k)}_{ij}$ is the k-step transition probability of τ_k from state i to j.

Proof: The comparison rule in (15) implies that the probability event $\{\tau^*_k = i\} \in \sigma(\tau_k, \tau_{k-1}, \ldots, \tau_1, \tau_0)$. Thus, it is readily concluded that τ^*_k is also a Markov chain since τ_k as a Markov chain evolves independently. It is obvious that τ^*_k can not be ∞ and thus its state space is $\mathcal{M}^* = \{0, 1, 2, \ldots, \bar{\tau}\}$. Furthermore, noticing $\{\tau^*_k = i\} = \{\tau^*_{k-1} = i - 1, \tau_k > i - 1\} \cup \{\tau^*_{k-1} \geq i, \tau_k = i\}$ we have

1. If $j \leq i$, then

$$\begin{aligned} P\{\tau^*_{k+1} = j | \tau^*_k = i\} &= P\{\tau_{k+1} = j | \tau^*_k = i\} \\ &= P\{\tau_{k+1} = j | \tau_k \geq i\} \\ &= \frac{\sum\limits_{l_1 \in \mathcal{M}, l_1 \geq i} \pi_{l_1}(k) \lambda_{l_1 j}}{\sum\limits_{l_1 \in \mathcal{M}, l_1 \geq i} \pi_{l_1}(k)} \end{aligned}$$

2. If $j = i + 1$, then

$$\begin{aligned} P\{\tau^*_{k+1} = j | \tau^*_k = i\} &= P\{\tau_{k+1} > i | \tau^*_k = i\} \\ &= P\{\tau_{k+1} > i | \tau_k \geq i\} \\ &= \frac{\sum\limits_{l_1 \in \mathcal{M}, l_1 \geq i} \sum\limits_{l_2 \in \mathcal{M}, l_2 > i} \pi_{l_1}(k) \lambda_{l_1 l_2}}{\sum\limits_{l_1 \in \mathcal{M}, l_1 \geq i} \pi_{l_1}(k)} \end{aligned}$$

which completes the proof.

The following well-known result for homogeneous ergodic Markov chains [27] is required for the stochastic stability analysis in this paper.

Lemma 4.2. *For the homogeneous ergodic Markov chain $\{\tau_k;\ k = 0, 1, \ldots\}$ with any initial distribution, there exists a limit probability distribution $\pi = \{\pi_i;\ \pi_i > 0,\ i \in \mathcal{M}\}$ such that for each $j \in \mathcal{M}$,*

$$\sum_{i \in \mathcal{M}} \lambda_{ij} \pi_i = \pi_j, \quad \sum_{i \in \mathcal{M}} \pi_i = 1 \tag{17}$$

and

$$|\pi_i(k) - \pi_i| \leq \eta \xi^k \tag{18}$$

for some $\eta \geq 0$ and $0 < \xi < 1$.

Proposition 4.1. *For N_1 that is large enough and some nonzero η^* the following inequality holds*

$$|\lambda^*_{ij}(k) - \lambda^*_{ij}| \leq \eta^* \xi^k, \quad k > N_1 \tag{19}$$

where $\Lambda^* = [\lambda^*_{ij}]$ with

$$\lambda^*_{ij} = \begin{cases} \dfrac{\sum\limits_{l_1 \in \mathcal{M}, l_1 \geq i} \pi_{l_1} \lambda_{l_1 j}}{\sum\limits_{l_1 \in \mathcal{M}, l_1 \geq i} \pi_{l_1}}, & \text{if } j \leq i; \\ \dfrac{\sum\limits_{l_1 \in \mathcal{M}, l_1 \geq i} \sum\limits_{l_2 \in \mathcal{M}, l_2 > i} \pi_{l_1} \lambda_{l_1 l_2}}{\sum\limits_{l_1 \in \mathcal{M}, l_1 \geq i} \pi_{l_1}}, & \text{if } j = i + 1; \\ 0, & \text{otherwise }. \end{cases} \tag{20}$$

Proof: It can be readily obtained from (16), (18) and (20).

4.2. **Stochastic stability and stabilization.** The following definition of stochastic stability is used in this paper.

Definition 4.1. *The closed-loop system in (14) is said to be stochastically stable if for every finite $X_0 = X(0)$ and initial state $\tau^*_0 = \tau^*(0) \in \mathcal{M}$, there exists a finite $W > 0$ such that the following inequality holds,*

$$E\left\{\sum_{k=0}^{\infty} ||X(k)||^2 | X_0, \tau^*_0\right\} < X_0^T W X_0 \tag{21}$$

where $E\{X\}$ is the expectation of the random variable X.

Theorem 4.1. *The closed-loop system in (14) is stochastically stable if and only if there exists $P(i) > 0$, $i \in \mathcal{M}^*$ such that the following $(\bar{\tau} + 1)$ LMIs hold*

$$L(i) = \sum_{j \in \mathcal{M}^*} \lambda^*_{ij} \Xi^T(j) P(j) \Xi(j) - P(i) < 0, \quad \forall i \in \mathcal{M}^* \tag{22}$$

Proof: Sufficiency. For the closed-loop system in (14), consider the following quadratic function given by

$$V(X(k),k) = X^T(k)P(\tau_k^*)X(k) \tag{23}$$

We have

$$\begin{aligned}
E\{\Delta V(X(k),k)\} &= E\left\{X^T(k+1)P(\tau_{k+1}^*)X(k+1)|X(k),\tau_k^* = i\right\} - X^T(k)P(i)X(k) \\
&= \sum_{j\in\mathcal{M}^*} \lambda_{ij}^*(k+1)X^T(k)\Xi^T(j)P(j)\Xi(j)X(k) - X^T(k)P(i)X(k) \\
&= X^T(k)\left[\sum_{j\in\mathcal{M}^*} \lambda_{ij}^*(k+1)\Xi^T(j)P(j)\Xi(j) - P(i)\right]X(k)
\end{aligned}$$

From condition (22) we obtain

$$X^T(k)\left[\sum_{j\in\mathcal{M}^*} \lambda_{ij}^*\Xi^T(j)P(j)\Xi(j) - P(i)\right]X(k) \le -\lambda_{\min}(-L(i))X^T(k)X(k) \le -\beta||X(k)||^2$$

where $\beta = \inf\{\lambda_{\min}(-L(i)); i\in\mathcal{M}^*\} > 0$. Thus, for $k > N_1$,

$$\begin{aligned}
E\{\Delta V(X(k),k)\} &= X^T(k)\left[\sum_{j\in\mathcal{M}^*} \lambda_{ij}^*(k+1)\Xi^T(j)P(j)\Xi(j) - P(i)\right]X(k) \\
&\le X^T(k)\left[\sum_{j\in\mathcal{M}^*} \lambda_{ij}^*\Xi^T(j)P(j)\Xi(j) - P(i)\right]X(k) \\
&\quad + X^T(k)\sum_{j\in\mathcal{M}^*} |\lambda_{ij}^*(k+1) - \lambda_{ij}^*|\Xi^T(j)P(j)\Xi(j)X(k) \\
&\le -\beta||X(k)||^2 + \eta^*\xi^{k+1}X^T(k)\sum_{j\in\mathcal{M}^*}\Xi^T(j)P(j)\Xi(j)X(k) \\
&\le \left(\alpha\eta^*\xi^{k+1} - \beta\right)||X(k)||^2
\end{aligned}$$

where $\alpha = \sup\{\lambda_{\max}(\Xi^T(j)P(j)\Xi(j)); j\in\mathcal{M}^*\} > 0$. Let $N_2 = \inf\{M; M\in\mathbb{N}^+, M > \max\{N_1, \log_\xi \frac{\beta}{\alpha\eta^*} - 1\}\}$. Then, we have for $k \ge N_2$.

$$E\{\Delta V(X(k),k)\} \le -\beta^*||X(k)||^2 \tag{24}$$

where $\beta^* = \beta - \alpha\eta^*\xi^{N_2+1} > 0$. Summing from N_2 to $N > N_2$, we obtain

$$\begin{aligned}
E\left\{\sum_{k=N_2}^{N}||X(k)||^2\right\} &\le \frac{1}{\beta^*}(E\{V(X(N_2),N_2)\} - E\{V(X(N+1),N+1)\}) \\
&\le \frac{1}{\beta^*}E\{V(X(N_2),N_2)\}
\end{aligned}$$

which implies that

$$E\left\{\sum_{k=0}^{\infty}||X(k)||^2\right\} \le \frac{1}{\beta^*}E\{V(X(N_2),N_2)\} + E\left\{\sum_{k=0}^{N_2-1}||X(k)||^2\right\} \tag{25}$$

This proves the stochastic stability of the closed-loop system in (14) by Definition 4.1.

Necessity. Suppose the closed-loop system in (14) is stochastically stable, that is,

$$E\left\{\sum_{k=0}^{\infty}||X(k)||^2|X_0,\tau_0^*\right\} < X_0^TWX_0 \tag{26}$$

Define

$$X^T(n)\bar{P}(N-n,\tau_n^*)X(n) = E\left\{\sum_{k=n}^{N} X^T(k)Q(\tau_k^*)X(k)|X_n,\tau_n^*\right\} \tag{27}$$

with $Q(\tau_k^*) > 0$. It is noticed that $X^T(n)\bar{P}(N-n,\tau_n^*)X(n)$ is upper bounded from (26) and monotonically non-decreasing as N increases since $Q(\tau_k^*) > 0$. Therefore, its limit exists which is denoted by

$$X^T(n)P(i)X(n) = \lim_{N\to\infty} X^T(n)\bar{P}(N-n,\tau_n^*=i)X(n) \tag{28}$$

Since (28) is valid for any $X(n)$, we obtain

$$P(i) = \lim_{N\to\infty} \bar{P}(N-n,\tau_n^*=i) > 0 \tag{29}$$

Now, consider

$$\begin{aligned} &E\left\{X^T(n)\bar{P}(N-n,\tau_n^*)X(n) - X^T(n+1)\bar{P}(N-n-1,\tau_{n+1}^*)X(n+1)|X_n,\tau_n^*=i\right\} \\ &= X^T(n)\left[\bar{P}(N-n,i) - \sum_{j\in\mathcal{M}^*}\lambda_{ij}^*(n+1)\Xi^T(j)\bar{P}(N-n-1,j)\Xi(j)\right]X(n) \\ &= X^T(n)Q(i)X(n) \end{aligned} \tag{30}$$

Since (30) is valid for any $X(n)$, we obtain

$$\bar{P}(N-n,i) - \sum_{j\in\mathcal{M}^*}\lambda_{ij}^*(n+1)\Xi^T(j)\bar{P}(N-n-1,j)\Xi(j)) = Q(i) > 0 \tag{31}$$

Let $N\to\infty$,

$$P(i) - \sum_{j\in\mathcal{M}^*}\lambda_{ij}^*(n+1)\Xi^T(j)P(j)\Xi(j) > 0, \quad \forall n$$

Let $n\to\infty$,

$$P(i) - \sum_{j\in\mathcal{M}^*}\lambda_{ij}^*\Xi^T(j)P(i)\Xi(j) > 0$$

which completes the proof.

The result below readily follows using the Schur complement.

Corollary 4.1. *System (1) is stochastically stabilizable using the packet-based control approach with the control law in (4) if and only if there exist* $P(i) > 0$, $Z(i) > 0$, $K(i)$, $i\in\mathcal{M}^*$ *such that the following* $(\bar{\tau}+1)$ *LMIs hold*

$$\begin{pmatrix} P(i) & R(i) \\ R^T(i) & Q \end{pmatrix} > 0, \quad i\in\mathcal{M}^* \tag{32}$$

with the equation constraints

$$P(i)Z(i) = I, \quad \forall i\in\mathcal{M}^* \tag{33}$$

where $R(i) = \left[(\lambda_{i0}^*)^{\frac{1}{2}}\Xi^T(0)\dots(\lambda_{i\bar{\tau}}^*)^{\frac{1}{2}}\Xi^T(\bar{\tau})\right]$, $Q = \mathrm{diag}\{Z(0)\dots Z(\bar{\tau})\}$ *and* $\Xi(i)$ *(consequently* $K(i)$*) is defined in (14).*

The LMIs in Corollary 4.1 with the matrix inverse constraints in (33) can be solved using the Cone Complementarity Linearization (CCL) algorithm [28].

5. **Illustrative Example.** A numerical example is considered in this section to illustrate the effectiveness of the propose approach in this paper.

Example 5.1. *Consider the system in (1) with the following system matrices borrowed from* [12]*,*

$$A = \begin{pmatrix} 1.0000 & 0.1000 & -0.0166 & -0.0005 \\ 0 & 1.0000 & -0.3374 & -0.0166 \\ 0 & 0 & 1.0996 & 0.1033 \\ 0 & 0 & 2.0247 & 1.0996 \end{pmatrix}, \quad B = \begin{pmatrix} 0.0045 \\ 0.0896 \\ -0.0068 \\ -0.1377 \end{pmatrix}.$$

This system is open-loop unstable with the eigenvalues at 1, 1, 1.5569 and 0.6423, respectively. In the simulation, the random round trip delay is upper bounded by 4, i.e., $\tau_k \in \mathcal{M} = \{0, 1, 2, 3, 4, \infty\}$*, with the following transition probability matrix,*

$$\Lambda = \begin{pmatrix} 0.1 & 0.2 & 0.2 & 0.3 & 0.2 & 0 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.1 & 0.1 \\ 0.24 & 0.06 & 0.48 & 0.12 & 0.1 & 0 \\ 0.15 & 0.25 & 0.3 & 0.15 & 0.1 & 0.05 \\ 0.3 & 0.3 & 0.2 & 0.1 & 0.1 & 0 \\ 0.3 & 0.3 & 0.15 & 0.15 & 0.1 & 0 \end{pmatrix}.$$

The limit distribution of the above ergodic Markov chain can be simply obtained by Lemma 4.2,

$$\pi = \begin{pmatrix} 0.1982 & 0.1814 & 0.3000 & 0.1738 & 0.1198 & 0.0268 \end{pmatrix}.$$

Λ^* *in Proposition 4.1 can then be calculated by (20) as*

$$\Lambda^* = \begin{pmatrix} 0.1982 & 0.8018 & 0 & 0 & 0 \\ 0.2224 & 0.1767 & 0.6008 & 0 & 0 \\ 0.2290 & 0.1699 & 0.3612 & 0.2398 & 0 \\ 0.2186 & 0.2729 & 0.2501 & 0.1313 & 0.1271 \\ 0.3000 & 0.3000 & 0.1909 & 0.1091 & 0.1000 \end{pmatrix}.$$

The comparison between the practical delays τ_k *and those after the comparison process using the packet-based control approach* τ_k^* *is illustrated in Figure 3 where 5 on the vertical axis represents a data packet dropout. From Figure 3, it is seen that data packet dropout has been effectively dealt with using the packet-based control approach, by noticing that* $\tau_k^* \in \mathcal{M}^* = \{0, 1, 2, 3, 4\}$*.*

From Corollary 4.1, the packet-based controller is obtained as follows, where it is seen that for different network conditions, different feedback gains are designed,

$$\begin{aligned} K(0) &= \begin{pmatrix} 0.5292 & 0.6489 & 22.4115 & 2.8205 \end{pmatrix}, \\ K(1) &= \begin{pmatrix} 0.3792 & 0.8912 & 20.2425 & 5.3681 \end{pmatrix}, \\ K(2) &= \begin{pmatrix} 0.0499 & 0.4266 & 15.6574 & 5.7322 \end{pmatrix}, \\ K(3) &= \begin{pmatrix} -0.4400 & -0.3003 & 9.2976 & 5.0540 \end{pmatrix}, \\ K(4) &= \begin{pmatrix} -0.8400 & -1.3422 & 2.7723 & 2.9173 \end{pmatrix}. \end{aligned}$$

Using the packet-based control approach with the above packet-based controller, the state trajectories of the closed-loop system is illustrated in Figure 4 with the initial states $x(-3) = x(-2) = x(-1) = x(0) = [0\ 0.1\ 0\ \ -0.1]^T$*, which demonstrates the stochastic stability of the closed-loop system.*

However, without the packet-based control strategy, even using the same controller design method without considering the different network conditions (that is, using $K(i) \equiv$

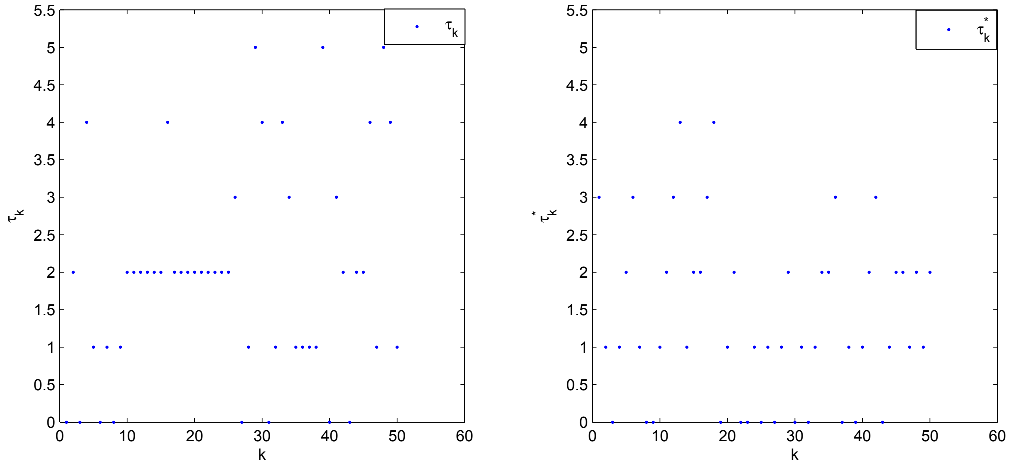

FIGURE 3. Comparison of the practical delays τ_k and those after the comparison process τ_k^* where 5 on the vertical axis represents a data packet dropout

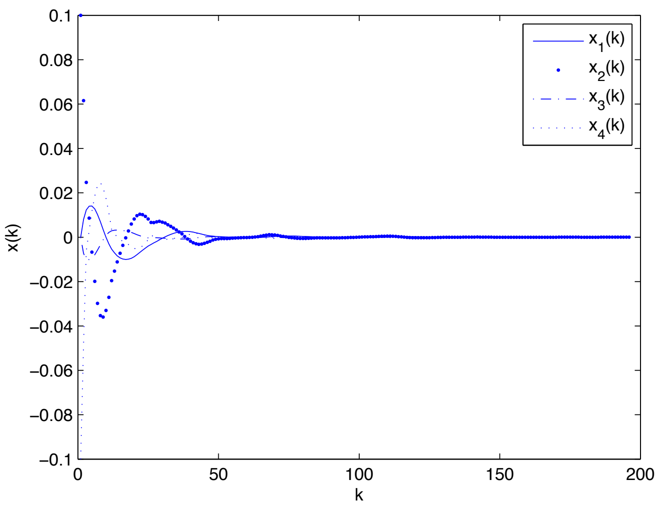

FIGURE 4. The system is stable using the packet-based control approach

$K(0)$, $i \in \mathcal{M}$, i.e., $K(0)$ fixed for all network conditions), the system is shown to be unstable under the same simulation conditions, see Figure 5. Furthermore, consider the conventional control approach proposed in [29] *where no packet-based control structure was considered and the feedback gain was designed as $K = [0.9844\ 1.6630\ 25.9053\ 6.1679]$ fixed for all network conditions, the system is also shown to be unstable under the same simulation conditions, see Figure 6. These comparisons proves the effectiveness of the proposed packet-based control approach and the stabilized controller design method.*

6. **Conclusion.** By taking advantage of the packet-based data transmission in NCSs, a packet-based control approach is proposed for NCSs, which can be used to deal with the communication constraints in NCSs including network-induced delay, data packet dropout and data packet disorder simultaneously. To the best of the authors' knowledge, this benefit has not been achieved using conventional control approaches to NCSs. The novel model obtained based on this approach offers the designers the freedom of designing

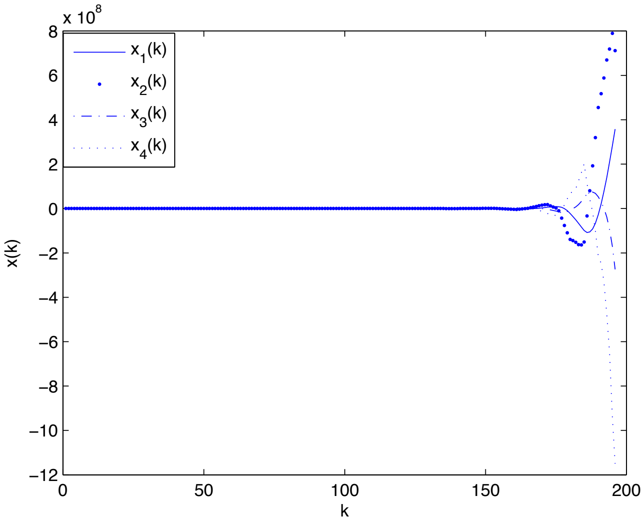

FIGURE 5. The system is unstable without the packet-based control strategy, using $K(0)$ fixed for all network conditions

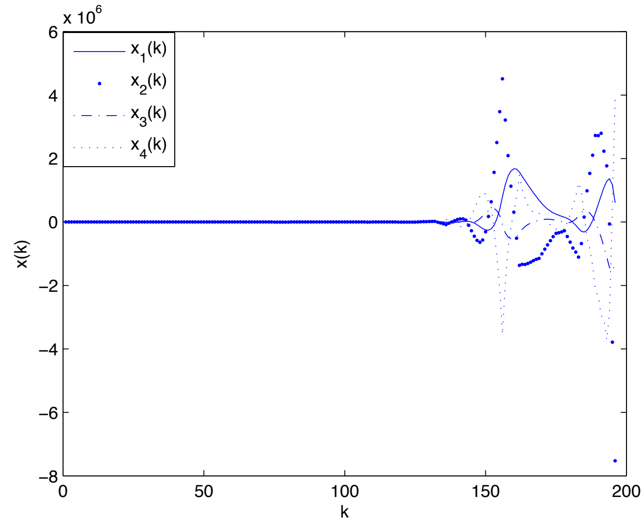

FIGURE 6. The system is unstable using conventional control approach with a fixed feedback gain

different controllers for different network conditions, which is realizable compared with the previous model in [12] where no supportive design method was considered and yet a similar model was used (Remark 2.1). The stochastic stabilization result is then obtained by modeling the communication constraints as a homogeneous ergodic Markov chain and then the closed-loop system as a Markov jump system. This result is based on a better understanding of the packet-based data transmission in the stochastic fashion and enabled the proposed packet-based control approach to be applied in practice. Within this packet-based control framework, future research will focus on nonlinear NCSs and experimental verification of the proposed approach.

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1. Stochastic stabilization of packet-based networked control systems

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