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SHORT PAPER

Stability Analysis of Nonlinear Switched Networked Control Systems with Periodical Packet Dropouts

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Abstract The input-to-state stability problem of a class of nonlinear switched networked control systems subject to time-varying transmission intervals, periodical packet dropouts, and communication constraints is investigated. By adopting the extended input-to-state stability(eISS) protocol and constructing a novel Lyapunov function, the input-to-state stability properties of such systems are discussed. Then, by making use of the small-gain theorem, the maximum allowable transmission interval to guarantee system stability is obtained. A batch reactor is finally considered to demonstrate the effectiveness of the proposed method.

Keywords Switched systems · Nonlinear networked control systems · Packet dropouts

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1 Introduction

Networked control systems (NCSs) are control systems where the control links are completed via communication networks. Despite the advantages of introducing communication networks into control systems, we have to deal with the imperfect data exchanges in NCSs before such systems can be widely deployed in practice [2, 5, 7, 13, 20, 22–24]. On the other hand, switched systems are a special type of hybrid dynamical systems which consist of finite subsystems and their dynamic behaviour is determined by a switching rule amongst the subsystems [8–10, 14, 18, 19]. The switching behaviour comes from either the controller design or the inherent system characteristics; examples can be seen in communication networks, computer synchronisation, traffic control, and so on [21, 25]. It is widely noticed that NCSs exhibit high switching behaviours and thus the design and analysis of NCSs within the switched system framework are highly desirable.

The effects of the imperfect data changes in NCSs can be roughly categorised as [1, 3]: (i) Quantisation errors; (ii) Packet dropouts; (iii) Variable sampling/ transmission intervals; (iv) Variable communication delays; (v) Communication constraints, i.e. not all sensor and actuator signals can be transmitted at the same time. With a nonlinear switched plant, the considered system in the present work is modelled as a nonlinear switched NCS which has not been addressed before. We are interested in the input-to-state stability (ISS) property [4] for the nonlinear switched NCS with the aforementioned network effects of categories (ii), (iii) and (v). Although this combination of the network effects has been considered before in [12], the conditions obtained there are difficult to verify in the case of packet dropout. In this work for the case of packet dropout, adopting the extended input-to-state stability(eISS) protocols in [15], we present a simple method to obtain the maximum allowable transmission interval (MATI). In our model, packet dropouts may take place periodically which is described by a periodical time sequence. What is more, in the considered hybrid systems the impulsive signals can have arbitrary jumps which is much more general than in the existing results [11].

The remainder of the paper is organised as follows. In Sect. 2, the considered model and the quoted protocol are given. The ISS for the hybrid system and the small-gain theorem for nonlinear NCSs are given in Sect. 3. In Sect. 4, a numerical example, batch reactor, is discussed. The conclusions are drawn in the last section.

Notations $\mathbb{R}$ and $\mathbb{N}$ denote the sets of the real numbers and nonnegative integers, respectively. $\mathbb{R}_{\geq 0} \triangleq [0, \infty)$, and $\mathbb{N}^+ \triangleq \mathbb{N}/\{0\}$. $|x|$ is the Euclid norm of $x \in \mathbb{R}^m$ and I_d is the identity function with dimension d. $\forall t \geq 0, 0 \leq t_1 \leq t_2$, $\|x[t_1, t_2]\| := \sup_{t_1 \leq t \leq t_2} |x(t)|$. For a matrix $A \in \mathbb{R}^{n \times n}$, $|A|$ is the induced matrix norm. A function $\alpha : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is said to be of class $\mathcal{K}$-function if it is continuous, zero at zero, and strictly increasing. It is a class $\mathcal{K}_\infty$-function if it is a class $\mathcal{K}$-function and unbounded. A function $\beta : \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \to \mathbb{R}^+$ is a class $\mathcal{KL}$-function if $\beta(\cdot, t)$ is a class $\mathcal{K}$-function for each $t \geq 0$ and $\beta(s, \cdot)$ is decreasing to zero for each $s \geq 0$. β is said to be a class of exp-$\mathcal{KL}$-function if there exist two positive constants M, λ such that $\beta(s, t) = Me^{-\lambda t}s$.

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2 The Model of NCSs and eISS Protocols

2.1 The Nonlinear Switched NCSs

In order to consider the systems in this paper, a monotonically increasing sequence of times t_ι, $\iota \in \mathbb{N}$ is given, where $t_0 = 0$, and $\lim_{\iota \to +\infty} t_\iota = +\infty$. Moreover, we assume that there exist ε and δ_ι such that

$$\text{(A0):} \quad 0 < \varepsilon \le t_\iota - t_{\iota-1} \le \delta_\iota \le \delta$$

which rules out the possibility of Zeno solutions.

Consider the following general switched plant without any network effects:

$$\dot{x}(t) = \tilde{f}_{\sigma(t)}(t, x, u, \omega), \tag{1}$$

where $x \in \mathbb{R}^{n_x}$, $u \in \mathbb{R}^{n_u}$ and $\omega \in \mathbb{R}^{n_\omega}$ are the system state, system input, and the disturbance input, respectively; $\sigma(t) : \mathbb{R}_{\ge 0} \to \{1, 2, \dots, n\}$ is the switching signal; $\tilde{f}_p : \mathbb{R}_{\ge 0} \times \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \times \mathbb{R}^{n_\omega} \to \mathbb{R}$, $p \in \{1, 2, \dots, n\}$ are smooth functions.

The match controller is

$$u(t) = k_{\sigma(t)}(t, x, \omega), \tag{2}$$

where $k_p : \mathbb{R}_{\ge 0} \times \mathbb{R}^{n_x} \times \mathbb{R}^{n_\omega} \to \mathbb{R}$, $p \in \{1, 2, \dots, n\}$ are smooth functions such that the switched system (1) with its controller (2) is stable. Under the effect of network, the state x is usually not directly available for the controller and thus the controller is given as

$$u(t) = k_{\sigma(t)}(t, \hat{x}, \omega), \tag{3}$$

where $\hat{x}$ is the vector of most recently transmitted system state via the network. We assume that the vector can be divided into l parts, $1 \le l \le n_x$, enumerated from 1 to l, and the ith sub-vector is referred as the ith node. At each transmission time t_ι, one of the nodes $i \in \{1, 2, \dots, l\}$ is granted the access to the network, and thus, $\hat{x}_i(t_\iota) = x_i(t_\iota)$. Here we assume that $\hat{x}$ is held constant between the transmission instants by using zero-order hold. We rewrite system (1) and (3) and obtain the general switched nonlinear NCSs of the following form:

$$\begin{aligned} &\dot{x}(t) = \tilde{f}_{\sigma(t)}\big(t, x, k_{\sigma(t)}(t, \hat{x}, \omega), \omega\big) \quad \forall t \in [t_{\iota-1}, t_\iota), \\ &\dot{\hat{x}}(t) = 0 \quad \forall t \in [t_{\iota-1}, t_\iota), \\ &\hat{x}(t_\iota) = x\big(t_\iota^-\big) + h\big(t_\iota^-, e\big), \end{aligned} \tag{4}$$

where $e \triangleq \hat{x} - x$ is the network-induced error. The function h is typical for time-scheduling protocols [12] and has the following form:

$$h\big(t_\iota^-, e\big) = \big(I - \Psi(s)\big)e\big(t_\iota^-\big), \tag{5}$$

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where $s = s(i, e) : \mathbb{N} \times \mathbb{R}^n \to \{1, 2, \ldots, l\}$ is some scheduling function

$$\Psi(s) := \mathrm{diag}\{\delta_{1_s} I_{n_1}, \ldots, \delta_{l_s} I_{n_l}\}, \tag{6}$$

where δ_{i_j} is the standard Kronecker delta and I_{n_j} is the identity matrix of dimension n_j, with $\sum_{j=1}^{l} n_j = n_e$. Based on $\hat{x}_i(t_\iota) = x_i(t_\iota)$, $e_i(t_\iota) = 0$ holds at time t_ι.

Rewriting the system (1), (3) and (5) in the (x, e) coordinates, we obtain the following NCSs model:

$$\dot{x}(t) = f_{\sigma(t)}(t, x, e, \omega), \quad t \in [t_{\iota-1}, t_\iota), \tag{7}$$

$$\dot{e}(t) = g_{\sigma(t)}(t, x, e, \omega), \quad t \in [t_{\iota-1}, t_\iota), \tag{8}$$

$$e(t_\iota) = h\big(t_\iota^-, e\big), \tag{9}$$

where

$$f_{\sigma(t)}(t, x, e, \omega) = \tilde{f}_{\sigma(t)}\big(t, x, k_{\sigma(t)}(t, x + e, \omega), \omega\big),$$
$$g_{\sigma(t)}(t, x, e, \omega) = -f_{\sigma(t)}(t, x, e, \omega)$$

and $x \in \mathbb{R}^{n_x}$, $e \in \mathbb{R}^{n_e}$ and $\omega \in \mathbb{R}^{n_\omega}$ are the state, the network-induced error and the disturbance input, respectively; $\sigma(t) : \mathbb{R}_{\geq 0} \to \{1, 2, \ldots, q\}$ is the switching signal; $f_p : \mathbb{R}_{\geq 0} \times \mathbb{R}^{n_x} \times \mathbb{R}^{n_e} \times \mathbb{R}^{n_\omega} \to \mathbb{R}$, $g_p : \mathbb{R}_{\geq 0} \times \mathbb{R}^{n_x} \times \mathbb{R}^{n_e} \times \mathbb{R}^{n_\omega} \to \mathbb{R}$, $p \in \{1, 2, \ldots, n\}$ and f_p and g_p are smooth functions. Here the bound δ in (A0) is the so-called MATI. With assumption (A0), for the solution to the above system, please refer to [12]. Also the following assumption is made:

Assumption 1 The considered system only switches at transmission times.

We use the following definition for subsystem (7).

Definition 1 [16] The subsystem (7) is said to be ISS from e and ω to x, if there exist a class $\mathcal{KL}$-function β and class $\mathcal{K}$-functions γ_{xe} and $\gamma_{x\omega}$ such that, for any initial state x_0 and any measurable, locally essentially bounded input e and disturbance input ω, the solution exists for all $t \geq 0$ and

$$\big|x(t)\big| \leq \beta(|x_0|, t) + \gamma_{xe}\big(\big\|e[0, t]\big\|\big) + \gamma_{x\omega}\big(\big\|\omega[0, t]\big\|\big). \tag{10}$$

If the system is ISS with linear functions γ_{xe} and $\gamma_{x\omega}$ and an exp-$\mathcal{KL}$-function $\beta(\cdot, \cdot)$, then we say that the system is ISS with linear gains and an exp-$\mathcal{KL}$-function.

2.2 The eISS Protocol

Unlike the general case of random packet dropouts considered in [12], we consider a special case where the packet dropouts occur periodically. Each period contains $m + n$ updating time instants, including two classes: the first m instants being transmitted without packet dropouts and the remaining n instants with packet dropouts. This periodical sequence is described by replacing $\{t_\iota\}$ with $\{t_\iota^k\}$ where $t_0^1 = t_0$,

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$\iota \in \{1, 2, \ldots, m+n\}$, and $k \in \mathbb{N}^+$ is the number of the period. Let $x(t_{m+n}^k) \triangleq x(t_0^{k+1})$ and denote the left limit of $x(t_\iota^k)$ by $x(t_\iota^{k-})$ such that the periodical time sequence satisfies (A0). Inspired by the ISS protocols [16], with packet dropouts and disturbance input, we rewrite (5) as

$$h\big(t_\iota^{k-}, e, \theta\big) = \big(I - \Psi(s)\big)e\big(t_\iota^{k-}\big) + \Psi(s)\theta\big(t_\iota^{k-}\big) \tag{11}$$

where $\theta(t_\iota^k) := \theta(t_\iota^k, x, \omega)$, and $\theta : \mathbb{N} \times \mathbb{R}^{n_x} \times \mathbb{R}^{n_\omega} \to \mathbb{R}^{n_e}$.

Replacing protocol (9) by (11), we have the following system:

$$\dot{x}(t) = f_{\sigma(t)}(t, x, e, \omega), \quad t \in \big[t_{\iota-1}^k, t_\iota^k\big), \tag{12}$$

$$\dot{e}(t) = g_{\sigma(t)}(t, x, e, \omega), \quad t \in \big[t_{\iota-1}^k, t_\iota^k\big), \tag{13}$$

$$e\big(t_\iota^k\big) = h\big(t_\iota^{k-}, e, \theta\big), \tag{14}$$

where $f_{\sigma(t)}(t, x, e, \omega)$ and $g_{\sigma(t)}(t, x, e, \omega)$ are defined above, and the parameters x, e, ω and $\sigma(t)$ are the same as in the system (7)–(9).

Definition 2 [15] Suppose that there exist a Lyapunov function $W(t, e) : \mathbb{R}_{\geq 0} \times \mathbb{R}^{n_e} \to \mathbb{R}_{\geq 0}$ and positive constants $\rho_1 < 1$, $\rho_2 \geq 1$, $\alpha_1, \alpha_2, G_1, G_2, G_3, G_4$, such that the following conditions hold for system (14) at each transmission time t_ι^k for all $e \in \mathbb{R}^{n_e}, x \in \mathbb{R}^{n_x}, \omega \in \mathbb{R}^{n_\omega}$

$$\alpha_1|e| \leq W(t, e) \leq \alpha_2|e|, \tag{15}$$

$$W\big(t_\iota^k, h\big(t_\iota^{k-}, e, \theta\big)\big) \leq \rho_1 W\big(t_\iota^{k-}, e\big) + G_3\big|x\big(t_\iota^{k-}\big)\big| + G_1\big|\omega\big(t_\iota^{k-}\big)\big|, \quad 0 < \iota \leq m, \tag{16}$$

$$W\big(t_\iota^k, h\big(t_\iota^{k-}, e, \theta\big)\big) \leq \rho_2 W\big(t_\iota^{k-}, e\big) + G_4\big|x\big(t_\iota^{k-}\big)\big| + G_2\big|\omega\big(t_\iota^{k-}\big)\big|, \quad m < \iota \leq m+n. \tag{17}$$

Then protocol (14) is said to be eISS with Lyapunov function $W(t, e)$.

Remark 1 The eISS protocol is equivalent to the ISS protocol in [16] if $n = 0$ and equivalent to the UGES protocol in [12] if $G_i = 0, i = \{1, 2, 3, 4\}$. Since the disturbance ω is involved in (16)–(17), this eISS protocol is more relaxed than the one considered in [12].

Consider the following non-switched system to demonstrate the concept of eISS:

$$\dot{x} = A_{11}x + A_{12}e + A_{13}\omega, \tag{18}$$

$$\dot{e} = A_{21}x + A_{22}e + A_{23}\omega, \tag{19}$$

$$e\big(t_\iota^k\big) = h\big(t_\iota^{k-}, e, x, \omega\big). \tag{20}$$

Example 1 der the try-once-discard (TOD) protocol [17]. In the TOD protocol, the scheduling function takes the following form:

$$s = s(e) = \min\Big\{\arg\max_i |e_i|\Big\}. \tag{21}$$

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The ith node has the greatest weighted error value and it will be granted access to the network at the time t_i^k. Let the Lyapunov function be $W(t, e) = |e|$, then we have $\alpha_1 = \alpha_2 = 1$. With the quantisation factor or the disturbance factor at the transmission time, (16) holds with some $G_1, G_3 \geq 0$, where $\rho_1 = \sqrt{\frac{l-1}{l}}$ [12]. At times due to the effect of packet dropouts, (17) may hold with $\rho_2 \geq 1$ and $G_2, G_4 \geq 0$ at some transmission time points. Thus the TOD protocol with some effect of packet dropouts is eISS with Lyapunov function $W(t, e) = |e|$.

Example 2 Consider the RR protocol, a static protocol [12, 17] where each node is granted access to the network in a given order without comparing the weighted value of each node. Let the Lyapunov function be

$$W(t, e) = \sqrt{\sum_{j=1}^{l} a_j^2(\iota)|e_j|^2},$$

where $a_j(\iota)$ are time varying coefficients and for any $\iota \in \mathbb{N}$ and any $j \in \{1, 2, \dots, l\}$ there exists a unique $d \in \{1, 2, \dots, l\}$ such that $a_j^2(\iota) = d$. We have $\alpha_1 = 1, \alpha_2 = \sqrt{l}$. Also, (16) holds with some $G_1, G_3 \geq 0$, where $\rho_1 = \sqrt{\frac{l-1}{l}}$ [12]. Due to the effect of packet dropout, (17) may hold with $\rho_2 \geq 1$ and $G_2, G_4 \geq 0$ at some transmission time points. Thus the RR protocol with some effect of packet dropouts is eISS with the Lyapunov function given above.

3 Stability Analysis

In this section, we consider the stability analysis problem of switched nonlinear NCSs (12)–(14), starting from the ISS property of the network-induced error system (13)–(14).

Lemma 1 *Suppose there exist a piecewise Lyapunov function $v(t, e) : \mathbb{R}_{\geq 0} \times \mathbb{R}^{n_e} \to \mathbb{R}_{\geq 0}$, class $\mathcal{K}$ functions $\alpha_1, \alpha_2, \chi_1, \chi_2$ and positive numbers $\rho_1 < 1, \rho_2 \geq 1, \mu$ such that for $e \in \mathbb{R}^{n_e}$ and $\theta(t_\iota^k) := \theta(t_\iota^k, x, \omega) \in \mathbb{R}^{n_e}$, $\bar{\theta}(t) \triangleq (x^T, \omega^T)^T \in \mathbb{R}^{n_x + n_\omega}$*

1. $$\alpha_1(|e|) \leq v(t, e) \leq \alpha_2(|e|); \tag{22}$$
2. $$\begin{cases} v(t_\iota^k, h(t_\iota^{k-}, e, \theta)) \leq \rho_1 v(t_\iota^{k-}, e) + \chi_2(|\theta(t_\iota^{k-})|) & 0 < \iota \leq m; \\ v(t_\iota^k, h(t_\iota^{k-}, e, \theta)) \leq \rho_2 v(t_\iota^{k-}, e) + \chi_2(|\theta(t_\iota^{k-})|) & m < \iota \leq m + n; \end{cases} \tag{23}$$
3. $$D^+ v(t, e) \leq \mu v(t, e) + \chi_1\left(\left|\bar{\theta}(t)\right|\right) \ \forall t \in \left[t_{\iota-1}^k, t_\iota^k\right); \tag{24}$$
4. $$\rho = \rho_1 \rho_2^n M^{n+1} \leq 1, \tag{25}$$

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where $D^+v(t,e)$ *denotes the right-hand derivative,* $\rho=\rho_1M(\rho_2M)^n$, $M=e^{\mu\delta}$, *and* $\gamma_0=\frac{1}{\mu}(M-1)$. *Then the following inequality holds for any* $t\in[t^k_{\iota-1},t^k_\iota)$, $0<\iota\le m$:

$$
\begin{aligned}
v(t,e) \le{}& \rho^{k-1}(\rho_1M)^\iota Mv(t_0,e_0)+\frac{\rho}{1-\rho}\sum_{l=0}^{m-1}(\rho_1M)^l\gamma_0\chi_1\big(\|\bar\theta[t_0,t)\|\big)\\
&+\sum_{s=1}^{k-1}\rho^{k-s}\sum_{l=1}^{m-1}(\rho_1M)^{m-1-l}M\chi_2\big(\big|\theta\big(t_l^{s-}\big)\big|\big)\\
&+\frac{1}{1-\rho}\sum_{j=0}^{n}(\rho_2M)^j\gamma_0\chi_1\big(\|\bar\theta[t_0,t)\|\big)\\
&+\sum_{s=0}^{k-1}\rho^{k-1-s}\sum_{j=0}^{n}(\rho_2M)^jM\chi_2\big(\big|\theta\big(t^{s-}_{m+n-j}\big)\big|\big)\\
&+\sum_{l=0}^{\iota-1}(\rho_1M)^l\gamma_0\chi_1\big(\|\bar\theta[t_0,t)\|\big)+\sum_{l=1}^{\iota-1}(\rho_1M)^{\iota-1-l}M\chi_2\big(\big|\theta\big(t_l^{k-}\big)\big|\big)
\end{aligned}
\tag{26}
$$

and the following inequality holds for any $t\in[t^k_{\iota-1},t^k_\iota)$, $m<\iota\le m+n$:

$$
\begin{aligned}
v(t,e) \le{}& \rho^{k-1}(\rho_2M)^{\iota-m-1}(\rho_1M)^mMv(t_0,e_0)\\
&+(\rho_2M)^{\iota-m-1}\frac{\rho}{1-\rho}\sum_{l=0}^{m-1}(\rho_1M)^l\gamma_0\chi_1\big(\|\bar\theta[t_0,t)\|\big)\\
&+(\rho_2M)^{\iota-m-1}\sum_{s=1}^{k-1}\rho^{k-s}\sum_{l=1}^{m-1}(\rho_1M)^{m-1-l}M\chi_2\big(\big|\theta\big(t_l^{s-}\big)\big|\big)\\
&+(\rho_2M)^{\iota-m-1}\frac{1}{1-\rho}\sum_{j=0}^{n}(\rho_2M)^j\gamma_0\chi_1\big(\|\bar\theta[t_0,t)\|\big)\\
&+(\rho_2M)^{\iota-m-1}\sum_{s=0}^{k-1}\rho^{k-1-s}\sum_{j=0}^{n}(\rho_2M)^jM\chi_2\big(\big|\theta\big(t^{s-}_{m+n-j}\big)\big|\big)\\
&+(\rho_2M)^{\iota-m-1}\sum_{l=0}^{\iota-1}(\rho_1M)^l\gamma_0\chi_1\big(\|\bar\theta[t_0,t)\|\big)\\
&+(\rho_2M)^{\iota-m-1}\sum_{l=1}^{\iota-1}(\rho_1M)^{\iota-1-l}M\chi_2\big(\big|\theta\big(t_l^{k-}\big)\big|\big)\\
&+\sum_{j=0}^{\iota-m-1}(\rho_2M)^j\gamma_0\chi_1\big(\|\bar\theta[t_0,t)\|\big)+\sum_{j=1}^{\iota-m-1}(\rho_2M)^jM\chi_2\big(\big|\theta\big(t^{k-}_{\iota-j}\big)\big|\big).
\end{aligned}
\tag{27}
$$

Proof See the Appendix. □

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Proposition 1 *Under the conditions of Lemma 1, system (13)–(14) is uniformly ISS with gain*

$$\gamma = \alpha_1^{-1} \circ (I+\varepsilon) \circ \left(\left[\rho \sum_{l=0}^{m-1} (\rho_1 M)^l + \sum_{j=0}^{n} (\rho_2 M)^j \right] \frac{1}{1-\rho} \gamma_0 \chi_1 + \left[\rho \sum_{l=1}^{m-1} (\rho_1 M)^{m-1-l} + \sum_{j=0}^{n} (\rho_2 M)^j \right] \frac{1}{1-\rho} M \chi_2 \right), \tag{28}$$

where ε is a class $\mathcal{K}$-function.

Proof See the Appendix. □

For linear systems (18)–(20), the following inequality where $\mu \geq 0$, $\chi_x, \chi_\omega \geq 0$, instead of (24), can be readily established:

$$D^+ W(t,e) \leq \mu W(t,e) + \chi_x |x| + \chi_\omega |\omega|. \tag{29}$$

For linear system (18)–(20), we will have the following proposition based on Proposition 1.

Proposition 2 *Given the Lyapunov function $W(t_\iota^k, e)$, suppose the following conditions hold*:

1. *The protocol (20) is eISS with Lyapunov function $W(t_\iota^k, e)$;*
2. *Along the error dynamics (19) for all t, x, ω and almost all e, (29) holds;*
3. *MATI satisfies $\delta \in (\varepsilon, T^*)$, where $T^* = \frac{1}{\mu(n+1)} \ln \frac{1}{\rho_2^n \rho_1}$.*

Then the ISS property holds for system (19)–(20), that is, for $t \in [t_{\iota-1}^k, t_\iota^k)$, $\iota \in \{1, 2, \ldots, m+n\}$, $k \in \mathbb{N}^+$,

$$\left|e(t)\right| \leq \beta\left(|e_0|, t-t_0\right) + \gamma_{ex} \left\|x[t_0, t]\right\| + \gamma_{e\omega} \left\|\omega[t_0, t]\right\|, \tag{30}$$

where

$$\beta\left(|e_0|, t-t_0\right) = \frac{\alpha_2}{\alpha_1} \rho^{-k(m+n)} M e^{\frac{t-t_0}{\delta} \ln \rho} |e_0|$$

$$\gamma_{ex} = \alpha_1^{-1} \left\{ \left[\rho \sum_{j=0}^{m-1} (\rho_1 M)^j + \sum_{j=0}^{n} (\rho_2 M)^j \right] \frac{1}{1-\rho} \gamma_0 \chi_x + \left[\rho \sum_{j=1}^{m-1} (\rho_1 M)^{m-1-j} + \sum_{j=0}^{n} (\rho_2 M)^j \right] \frac{1}{1-\rho} M G \right\}$$

$$\gamma_{e\omega} = \alpha_1^{-1} \left\{ \left[\rho \sum_{j=0}^{m-1} (\rho_1 M)^j + \sum_{j=0}^{n} (\rho_2 M)^j \right] \frac{1}{1-\rho} \gamma_0 \chi_\omega + \left[\rho \sum_{j=1}^{m-1} (\rho_1 M)^{m-1-j} + \sum_{j=0}^{n} (\rho_2 M)^j \right] \frac{1}{1-\rho} M G \right\},$$

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$\rho = \rho_1 M(\rho_2 M)^n$, $M = e^{\mu\delta}$, $\gamma_0 = \frac{1}{\mu}(M-1)$, $G = \max\{G_1, G_2, G_3, G_4\}$ and $G_i, i \in \{1, 2, 3, 4\}$, α_1, α_2, ρ_1 and ρ_2 are defined in Definition 2.

Proof It can be readily obtained from Lemma 1 and Proposition 1. □

Example 3 For the TOD protocol, consider the Lyapunov function $W(t, e) = |e|$. From Example 1, Condition 1 in Proposition 2 holds with $\alpha_1 = \alpha_2 = 1$, $\rho_1 = \sqrt{\frac{l-1}{l}}$, $\rho_2 \geq 1$ and $\chi_2 = \max\{G_i\}$, $i = \{1, 2, 3, 4\}$. Based on the analysis of [12], we have $\mu = \frac{1}{2}\lambda_{\max}(A_{22}^T + A_{22})$, $\chi_x = |A_{21}|$ and $\chi_\omega = |A_{23}|$. Thus Condition 2 in Proposition 2 holds. Therefore, if Condition 3 holds, then the system (19)–(20) is ISS with γ_{ex} and $\gamma_{e\omega}$ defined in Proposition 2.

Example 4 Let the Lyapunov function for the RR protocol be

$$W(t, e) = \sqrt{\sum_{j=1}^{l} a_j^2(\iota)|e_j|^2}.$$

We have $\alpha_1 = 1$, $\alpha_2 = \sqrt{l}$, $\rho_1 = \sqrt{\frac{l-1}{l}}$, $\rho_2 \geq 1$ and $\chi_2 = \max\{G_i\}$, $i = \{1, 2, 3, 4\}$. Then Condition 1 in Proposition 2 holds. Similarly, $\mu = \sqrt{l}|A_{22}|$ and $\chi_x = \sqrt{l}|A_{21}|$, $\chi_\omega = \sqrt{l}|A_{23}|$, and Condition 2 in Proposition 2 holds. If Condition 3 holds, then the system (19)–(20) is ISS with γ_{ex} and $\gamma_{e\omega}$ defined in Proposition 2.

Theorem 1 *For the system (12)–(14), suppose the following conditions hold:*

1. *Subsystem (14) satisfies the following:*

$$\big|x(t)\big| \leq \beta_x\big(|x_0|, t - t_0\big) + \gamma_{xe}\big\|e[t_0, t]\big\| + \gamma_{x\omega}\big\|\omega[t_0, t]\big\|, \tag{31}$$

   where $\gamma_{xe} \geq 0$, $\gamma_{x\omega} \geq 0$;
2. *Conditions 1–3 in Proposition 2 hold;*
3. *There exists a MATI δ such that $\gamma_{xe} \times \gamma_{ex} < 1$, where γ_{xe} is defined in Definition 1 and γ_{ex} is equal to γ in Proposition 1.*

   Then the whole system (12)–(14) is ISS from the input signal ω with MATI δ.

Remark 2 This theorem is similar to the main results in [12]. The proof of this theorem follows readily from Theorem 2.1 in [6]. The gain γ_{ex} in Condition 2 can be easily obtained in the same way as in Lemma 1 and Proposition 1. It is pointed out that linear gains γ_{xe} and $\gamma_{x\omega}$ in (27) are not necessary, which can be relaxed by two class $\mathcal{K}$ functions.

For the non-switched system considered in [12],

$$\dot{x}(t) = f(t, x, e), \tag{32}$$

$$\dot{e}(t) = g(t, x, e), \tag{33}$$

$$e\big(t_\iota^k\big) = h\big(t_\iota^{k-}, e\big), \tag{34}$$

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where protocol (34) has the form of (11), and f and g are defined in system (12)–(13), we have the following corollary.

Corollary 1 *For the system* (32)–(34), *suppose the following conditions hold*:

1. *Subsystem* (32) *is ISS from e to x with gain* γ_{xe}, *where* $\gamma_{xe} \geq 0$;
2. *All the conditions in Lemma* 1 *hold, and subsystem* (33)–(34) *is ISS from x to e with gain* γ_{ex}, *where* γ_{ex} *has the same form as* γ *in Proposition* 1;
3. *There exists MATI* δ *such that* $\gamma_{xe} \times \gamma_{ex} < 1$.

Then the whole system (32)–(34) *is UGES.*

Remark 3 The following procedure is used to obtain the MATI.

(i) Calculate the ISS gain γ_{xe} from e to x with the parameters in subsystem (12);
(ii) Calculate T^* with the parameters in subsystem (13)–(14) and Condition 3 in Proposition 2 such that subsystem (13)–(14) is ISS from x to e;
(iii) Check Condition 3 in Theorem 1;
(iv) If it does not hold, use a new bound $T = T^* - \eta$, with $0 < \eta < T^*$ to calculate the ISS gain γ_{ex} from x to e. Then go to (iii);
(v) If Condition 3 in Theorem 1 holds, let the MATI $\delta = T$.

4 Numerical Example: The Batch Reactor

Consider the batch reactor reported in [12, 17]. The linearised model of the unstable batch reactor is of the following form in the presence of the communication network:

$$\dot{x}(t) = A_{11}x(t) + A_{12}e(t), \quad \forall t \in \left[t_{\iota-1}^k, t_\iota^k\right), \tag{35}$$

$$\dot{e}(t) = A_{21}x(t) + A_{22}e(t), \quad \forall t \in \left[t_{\iota-1}^k, t_\iota^k\right), \tag{36}$$

$$e\left(t_\iota^k\right) = h\left(t_\iota^{k-}, e\right), \tag{37}$$

where

$$A_{11} = \begin{pmatrix} 1.38 & -0.2077 & 6.7150 & -5.6760 & 0 & 0 \\ -0.5814 & -15.6480 & 0 & 0.6750 & -11.3580 & 0 \\ -14.6630 & 2.0010 & -22.3840 & 21.6230 & -2.2720 & -25.1680 \\ 0.048 & 2.0010 & 1.3430 & -2.1040 & -2.2720 & 0 \\ 0 & 1.0000 & 0 & 0 & 0 & 0 \\ 1.0000 & 0 & 1.0000 & -1.0000 & 0 & 0 \end{pmatrix};$$

$$A_{12} = \begin{pmatrix} 0 & 0 \\ 0 & -11.3580 \\ -15.7300 & -2.2720 \\ 0 & -2.2720 \\ 0 & 1.0000 \\ 1.0000 & 0 \end{pmatrix};$$

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$$A_{21} = \begin{pmatrix} 13.3310 & 0.2077 & 17.0120 & -18.0510 & 0 & 25.1680 \\ 0.5814 & 15.6480 & 0 & -0.6750 & 11.3580 & 0 \end{pmatrix};$$

$$A_{22} = \begin{pmatrix} 15.7300 & 0 \\ 0 & 11.3580 \end{pmatrix}.$$

Only the plant outputs are sent over the network in the above system. For (35), the gain from e to x is $\gamma_{xe} = 17.7653$.

Considering the eISS protocol and with the TOD form [17], let the Lyapunov function be $v(t,e) = |e|$. Then we have $\mu = 15.73$, $\rho_1 = \sqrt{\frac{l-1}{l}}$, where $l = 2$ is the number of the nodes. Then we choose $\rho_2 = 1$ and $m = n = 1$ which means that packet dropout occurs once after two adjacent samples.

With Condition 3 in Proposition 2, it is obtained that $T^* = 0.011$ s. Then with Condition 3 in Theorem 1, we have $\delta = 0.00017$ s, and the gain from x to e is $\gamma_{ex} = 0.056$. Then the whole system is UGES.

With the analysis in Example 2, let the Lyapunov function be

$$v(t,e) = \sqrt{\sum_{j=1}^{l} a_j^2(\iota)|e_j|^2}.$$

Similarly, we have that $\mu = 15.73$, $\rho_1 = 0.707$. Then we choose $\rho_2 = 1.1$ and $m = n = 1$. Ignoring the term of ω in Proposition 2, we obtain that $T^* = 0.008$ s. Then with Theorem 1, we have $\delta = 0.000079$ s, and the gain from x to e is $\gamma_{ex} = 0.056$. The whole system is UGES.

MATI that ensures the UGES of the system is calculated. Figure 1 shows the tradeoff curve between the MATI and the ratio of the number of normal transmission m and the number of corresponding packet dropouts n in a period. When $m : n = 99 : 1$, MATI is 0.000701 s and 0.000497 s for the TOD protocol and the RR protocol, re-

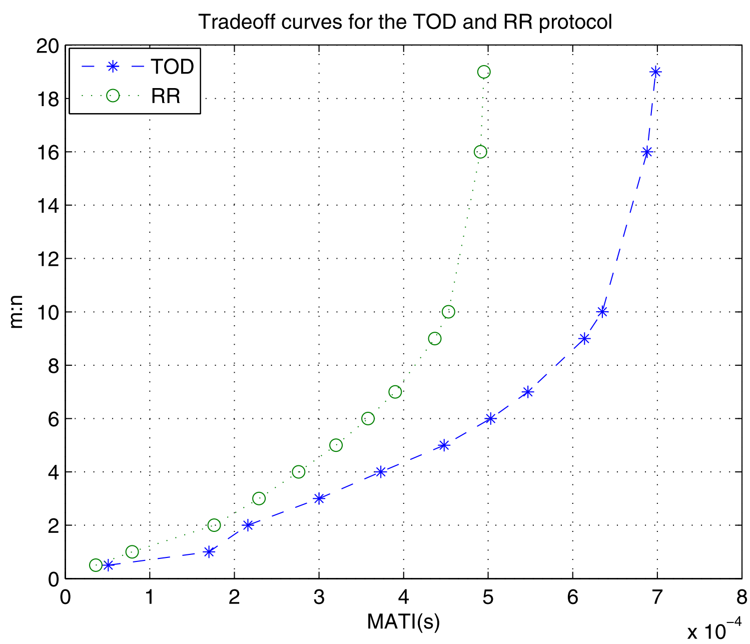

Fig. 1 Tradeoff curves for TOD and RR protocols

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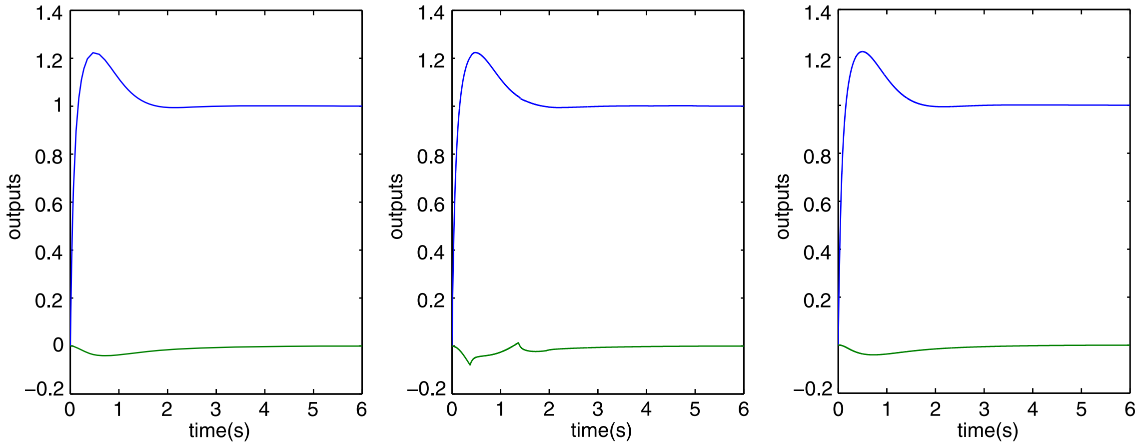

Fig. 2 The state responses of the batch process. The *first diagram* shows the response to follow $(y_1 y_2) = (10)$, the *second* is with TOD protocol and packet dropouts, and the *third* is with RR protocol and packet dropouts

spectively. The TOD protocol outperforms the RR protocol in the sense that the former allows larger transmission intervals. The step response of the batch reactor is illustrated in Fig. 2. It is shown that the system is stable under the conditions given in this paper, which validates the proposed approach.

5 Conclusions

A class of switched nonlinear NCSs with periodical packet dropouts, variable sampling intervals and communication constraints is investigated. With a novel piecewise Lyapunov function, the ISS properties for this model with protocols containing periodical packet dropouts are discussed. The small gain theorem is used to give the ISS conditions for the NCSs. Finally, the effectiveness of the proposed method is illustrated by a batch reactor example. Future research will focus on the use of multiple-Lyapunov method which is usually less conservative than the common Lyapunov function method used in the present work.

Appendix

Proof of Lemma 1 We use mathematical induction to prove (26) and (27). Considering Conditions 2–3, we can derive the following inequalities with the Lyapunov function $v(t, e)$ and we use $v(t)$ to denote $v(t, e)$ for ellipsis. For $t \in [t_\iota^k, t_{\iota+1}^k)$, we obtain that

$$v(t) \leq M v\big(t_\iota^k\big) + \gamma_0 \chi_1\big(\big\|\bar{\theta}[t_0, t)\big\|\big). \tag{38}$$

It holds for $0 < \iota \leq m$ that

$$v\big(t_{\iota+1}^k\big) \leq \rho_1 v\big(t_{\iota+1}^{k-}\big) + \chi_2\big(\big|\theta\big(t_{\iota+1}^{k-}\big)\big|\big). \tag{39}$$

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Inequalities (38) and (39) then imply

$$v(t_{\iota+1}^{k}) \le \rho_1\big[Mv(t_\iota^k)+\gamma_0\chi_1\big(\|\bar{\theta}[t_0,t)\|\big)\big]+\chi_2\big(\big|\theta\big(t_{\iota+1}^{k-}\big)\big|\big)$$
$$\le \rho_1 Mv(t_i^k)+\rho_1\gamma_0\chi_1\big(\|\bar{\theta}[t_0,t)\|\big)++\chi_2\big(\big|\theta\big(t_{\iota+1}^{k-}\big)\big|\big). \tag{40}$$

For $m<\iota\le m+n$ it holds that

$$v(t_{\iota+1}^{k}) \le \rho_2 v\big(t_{\iota+1}^{k-}\big)+\chi_2\big(\big|\theta\big(t_{\iota+1}^{k-}\big)\big|\big) \tag{41}$$

and

$$v(t_{\iota+1}^{k}) \le \rho_2\big[Mv(t_\iota^k)+\gamma_0\chi_1\big(\|\bar{\theta}[t_0,t)\|\big)\big]+\chi_2\big(\big|\theta\big(t_{\iota+1}^{k-}\big)\big|\big)$$
$$\le \rho_2 Mv(t_\iota^k)+\rho_2\gamma_0\chi_1\big(\|\bar{\theta}[t_0,t)\|\big)+\chi_2\big(\big|\theta\big(t_{\iota+1}^{k-}\big)\big|\big). \tag{42}$$

For $t\in[t_0^1,t_1^1)$,

$$v(t)\le Mv(t_0)+\gamma_0\chi_1\big(\|\bar{\theta}[t_0,t)\|\big) \tag{43}$$

and

$$v(t_1^1)\le \rho_1 Mv(t_0)+\rho_1\gamma_0\chi_1\big(\|\bar{\theta}[t_0,t)\|\big)+\chi_2\big(\big|\theta\big(t_1^{1-}\big)\big|\big). \tag{44}$$

Then for any $1<\iota\le m, k=1$, we have that

$$v(t)\le M(\rho_1 M)^{\iota-1}v(t_0)+\sum_{l=0}^{\iota-1}(\rho_1 M)^l\gamma_0\chi_1\big(\|\bar{\theta}[t_0,t)\|\big)$$
$$+\sum_{l=1}^{\iota-1}(\rho_1 M)^{\iota-1-l}M\chi_2\big(\big|\theta\big(t_l^{1-}\big)\big|\big). \tag{45}$$

For $t\in[t_m^1,t_{m+1}^1)$,

$$v(t)\le M(\rho_1 M)^{m}v(t_0)+\sum_{l=0}^{m}(\rho_1 M)^l\gamma_0\chi_1\big(\|\bar{\theta}[t_0,t)\|\big)$$
$$+\sum_{l=1}^{m}(\rho_1 M)^{m-l}M\chi_2\big(\big|\theta\big(t_l^{1-}\big)\big|\big). \tag{46}$$

With Condition 2, we obtain that

$$v(t_{m+1}^1)\le \rho_2 M(\rho_1 M)^{m}v(t_0)+\rho_2\sum_{l=0}^{m}(\rho_1 M)^l\gamma_0\chi_1\big(\|\bar{\theta}[t_0,t)\|\big)$$
$$+\rho_2\sum_{l=1}^{m}(\rho_1 M)^{m-l}M\chi_2\big(\big|\theta\big(t_l^{1-}\big)\big|\big)+\chi_2\big(\big|\theta\big(t_{m+1}^{1-}\big)\big|\big). \tag{47}$$

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For $\forall t \in [t_\iota^1, t_{\iota+1}^1)$, $m < \iota \leq m + n$, it holds that

$$\begin{aligned}
v(t) \leq\ & M(\rho_2 M)^{\iota-m}(\rho_1 M)^m v(t_0) + (\rho_2 M)^{\iota-m-1} \sum_{l=0}^{m} (\rho_1 M)^l \gamma_0 \chi_1\big(\|\bar{\theta}[t_0, t)\|\big) \\
& + (\rho_2 M)^{\iota-m-1} \sum_{l=1}^{m} (\rho_1 M)^{m-l} M \chi_2\big(|\theta(t_l^{1-})|\big) \\
& + \sum_{j=0}^{\iota-m-1} (\rho_2 M)^j \gamma_0 \chi_1\big(\|\bar{\theta}[t_0, t)\|\big) + \sum_{j=1}^{\iota-m-1} (\rho_2 M)^j M \chi_2\big(|\theta(t_{\iota-j}^{1-})|\big). \quad (48)
\end{aligned}$$

Then

$$\begin{aligned}
v(t_0^2) =\ & v(t_{m+n}^1) \\
\leq\ & (\rho_2 M)^n (\rho_1 M)^m v(t_0) + \rho_2(\rho_2 M)^{n-1} \sum_{l=0}^{m} (\rho_1 M)^l \gamma_0 \chi_1\big(\|\bar{\theta}[t_0, t)\|\big) \\
& + \rho_2(\rho_2 M)^{n-1} \sum_{l=1}^{m} (\rho_1 M)^{m-l} M \chi_2\big(|\theta(t_l^{1-})|\big) \\
& + \rho_2 \sum_{j=0}^{n-1} (\rho_2 M)^j \gamma_0 \chi_1\big(\|\bar{\theta}[t_0, t)\|\big) + \sum_{j=0}^{n-1} (\rho_2 M)^j M \chi_2\big(|\theta(t_{m+n-j}^{1-})|\big). \quad (49)
\end{aligned}$$

For $\forall t \in [t_0^2, t_1^2)$,

$$\begin{aligned}
v(t) \leq\ & (\rho_2 M)^n (\rho_1 M)^m M v(t_0) + (\rho_2 M)^n (\rho_1 M) \sum_{l=0}^{m-1} (\rho_1 M)^l \gamma_0 \chi_1\big(\|\bar{\theta}[t_0, t)\|\big) \\
& + (\rho_2 M)^n (\rho_1 M) \sum_{l=1}^{m-1} (\rho_1 M)^{m-1-l} M \chi_2\big(|\theta(t_l^{1-})|\big) \\
& + \sum_{j=0}^{n} (\rho_2 M)^j \gamma_0 \chi_1\big(\|\bar{\theta}[t_0, t)\|\big) + \sum_{j=0}^{n} (\rho_2 M)^j M \chi_2\big(|\theta(t_{m+n-j}^{1-})|\big). \quad (50)
\end{aligned}$$

With the definition $\rho = (\rho_2 M)^n \rho_1 M$ and $\rho_1 M < 1$, using mathematical induction, $\forall k \in \mathbb{N}^+$ and $0 < \iota \leq m$, it holds that

$$\begin{aligned}
v(t) \leq\ & \rho^{k-1}(\rho_1 M)^\iota M v(t_0) + \frac{\rho}{1-\rho} \sum_{l=0}^{m-1} (\rho_1 M)^l \gamma_0 \chi_1\big(\|\bar{\theta}[t_0, t)\|\big) \\
& + \sum_{s=1}^{k-1} \rho^{k-s} \sum_{l=1}^{m-1} (\rho_1 M)^{m-1-l} M \chi_2\big(|\theta(t_l^{s-})|\big) \\
& + \frac{1}{1-\rho} \sum_{j=0}^{n} (\rho_2 M)^j \gamma_0 \chi_1\big(\|\bar{\theta}[t_0, t)\|\big)
\end{aligned}$$

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$$+\sum_{s=0}^{k-1}\rho^{k-1-s}\sum_{j=0}^{n}(\rho_2 M)^j M\chi_2\big(\big|\theta\big(t_{m+n-j}^{s-}\big)\big|\big)$$

$$+\sum_{l=0}^{\iota-1}(\rho_1 M)^l\gamma_0\chi_1\big(\|\bar{\theta}[t_0,t)\|\big)+\sum_{l=1}^{\iota-1}(\rho_1 M)^{\iota-1-l}M\chi_2\big(\big|\theta\big(t_l^{k-}\big)\big|\big). \quad (51)$$

Then, $\forall k\in\mathbb{N}^+$ and $m<\iota\le m+n$,

$$\begin{aligned}v(t) &\le \rho^{k-1}(\rho_2 M)^{\iota-m-1}(\rho_1 M)^m M v(t_0)\\
&+(\rho_2 M)^{\iota-m-1}\frac{\rho}{1-\rho}\sum_{l=0}^{m-1}(\rho_1 M)^l\gamma_0\chi_1\big(\|\bar{\theta}[t_0,t)\|\big)\\
&+(\rho_2 M)^{\iota-m-1}\sum_{s=1}^{k-1}\rho^{k-s}\sum_{l=1}^{m-1}(\rho_1 M)^{m-1-l}M\chi_2\big(\big|\theta\big(t_l^{s-}\big)\big|\big)\\
&+(\rho_2 M)^{\iota-m-1}\frac{1}{1-\rho}\sum_{j=0}^{n}(\rho_2 M)^j\gamma_0\chi_1\big(\|\bar{\theta}[t_0,t)\|\big)\\
&+(\rho_2 M)^{\iota-m-1}\sum_{s=0}^{k-1}\rho^{k-1-s}\sum_{j=0}^{n}(\rho_2 M)^j M\chi_2\big(\big|\theta\big(t_{m+n-j}^{s-}\big)\big|\big)\\
&+(\rho_2 M)^{\iota-m-1}\sum_{l=0}^{i-1}(\rho_1 M)^l\gamma_0\chi_1\big(\|\bar{\theta}[t_0,t)\|\big)\\
&+(\rho_2 M)^{\iota-m-1}\sum_{l=1}^{i-1}(\rho_1 M)^{\iota-1-l}M\chi_2\big(\big|\theta\big(t_l^{k-}\big)\big|\big)\\
&+\sum_{j=0}^{\iota-m-1}(\rho_2 M)^j\gamma_0\chi_1\big(\|\bar{\theta}[t_0,t)\|\big)+\sum_{j=1}^{\iota-m-1}(\rho_2 M)^j M\chi_2\big(\big|\theta\big(t_{\iota-j}^{k-}\big)\big|\big), \quad (52)\end{aligned}$$

where $M=e^{\mu\delta}$, $\gamma_0=\frac{1}{\mu}(M-1)$. $\square$

Proof of Proposition 1 With (A0), $\forall t\in[t_{\iota-1}^k,t_\iota^k)$, $k\in\mathbb{N}^+$ and $0<\iota\le m+n$ we have that $t-t_0\le(k+1)(m+n)\delta$. Then with $\rho<1$, $\rho^{k+1}\le\rho^{\frac{t-t_0}{\delta(m+n)}}=e^{\frac{t-t_0}{\delta(m+n)}\ln\rho}$. Suppose that $|\theta(t_\iota)|\le\|\bar{\theta}[t_0,t)\|$, then with (51) and (52) the following inequality holds:

$$\begin{aligned}v(t)\le e^{\frac{t-t_0}{\delta(m+n)}\ln\rho}v(t_0)&+\left[\rho\sum_{l=0}^{m-1}(\rho_1 M)^l+\sum_{j=0}^{n}(\rho_2 M)^j\right]\frac{1}{1-\rho}\gamma_0\chi_1\big(\|\bar{\theta}[t_0,t)\|\big)\\
&+\left[\rho\sum_{l=1}^{m-1}(\rho_1 M)^{m-1-l}+\sum_{j=0}^{n}(\rho_2 M)^j\right]\frac{1}{1-\rho}M\chi_2\big(\|\bar{\theta}[t_0,t)\|\big). \quad (53)\end{aligned}$$

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Recalling Condition 1, we obtain

$$\begin{aligned}
\left|e(t)\right| &\leq \alpha_1^{-1} \circ \left(I + \varepsilon^{-1}\right) \circ e^{\frac{t-t_0}{\delta(m+n)} \ln \rho} v(t_0) \\
&\quad + \alpha_1^{-1} \circ (I + \varepsilon) \circ \left(\left[\rho \sum_{l=0}^{m-1} (\rho_1 M)^l + \sum_{j=0}^{n} (\rho_2 M)^j \right] \frac{1}{1-\rho} \gamma_0 \chi_1 \left(\left\| \bar{\theta}[t_0, t) \right\| \right) \right. \\
&\quad \left. + \left[\rho \sum_{l=1}^{m-1} (\rho_1 M)^{m-1-l} + \sum_{j=0}^{n} (\rho_2 M)^j \right] \frac{1}{1-\rho} M \chi_2 \left(\left\| \bar{\theta}[t_0, t) \right\| \right) \right). \qquad (54)
\end{aligned}$$

□

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