Asian Journal of Control, Vol. 20, No. 6, pp. 1–11, November 2018
Published online in Wiley Online Library (wileyonlinelibrary.com) DOI: 10.1002/asjc.1761

DELAYED FEEDBACK MPC ALGORITHMS OF VEHICLE PLATOONS SUBJECT TO CONSTRAINTS ON MEASUREMENT RANGE AND DRIVING BEHAVIORS

Shi-Ming Yu, Sai-Nan Wu, Yun-Bo Zhao and De-Feng He

ABSTRACT

The control problem of vehicle platoons considering sensors with limited measurement range and actuator time delay is investigated in the face of constraints. A new delayed feedback model predictive control scheme is proposed to solve the problem while satisfying the constraints on measurement range and driving behaviors. A family of controllers is presented with free parameters which are then computed by online solving of a receding horizon optimal control problem. Terminal equality constraints are adopted to guarantee stability of the closed-loop system. Some sufficient conditions with guaranteed string stability of the platoon and zero steady-state error are established. The effectiveness and advantages of the presented method are demonstrated by simulating two classical road scenarios.

Key Words: Vehicle platoons, model predictive control, adaptive cruise control, constraints, string stability.

I. INTRODUCTION

As one of the key technologies in the next generation of transportation, the automated highway/vehicle system has been attracting research in recent years [1]. In such a system one key concept is so-called vehicle platoons, where a number of vehicles are driven in one direction. While the first vehicle in vehicle platoons follows a reference trajectory, the remaining vehicles aim to keep some minimum safe distance to neighboring vehicles in order to increase the capacity of roads and the fuel economy of the platoon system [2–4]. As a result, road congestion, as well as traffic accidents, can be reduced significantly since the system eliminates the potential for human error [5].

It is well known that the smooth traffic flow of vehicle platoons is guaranteed by establishing the string stability of the platoon, which represents its ability to attenuate velocity fluctuation coming from the vehicles in front [6,7]. Past research has shown that string stability of vehicle platoons may be degraded due to the existence of time-delay for the actuator, sensor failure, and so on [8]. Hence, many efforts have been made to maintain string stability in the entire velocity range of vehicles [9–15]. For example, in [12] the effects of the parasitic delay on string stability of platoons was discussed. Then a sufficient condition was provided to achieve string stability of the platoon equipped with an adaptive cruise control (ACC) system. Moreover, the cooperative ACC system, which is an extension of the ACC system, was addressed to improve prediction ability and string stability of the platoon [16]. The authors in [13] presented some frequency-domain conditions for string stability of vehicle platoons by taking account of the time delays resulting from communication transmission and actuators. In [14,15] the sliding-model control was used to ensure string stability of vehicle platoons and to cooperatively track speed and acceleration of the preceding vehicles in the presence of the actuator time delay of vehicles.

In vehicle platoons, on-board cameras or laser sensors are used to measure the velocity, distance, and position of the preceding and surrounding vehicles. These vehicle sensors are subject to limited sensory capability in practical applications and hence reliably give measurement information only in a certain range. For instance, a radar sensor measuring the inter-distance of two vehicles often has a detection range, that is, the interval between the minimum and maximum detection distances, and a non-sensitive zone beyond the detection capability [17]. The limited sensory capability of on-board sensors has negative effects on the string stability of vehicle platoons [18]. To reduce these negative effects caused by the limited sensory capability, in [19–21] the H_∞ control method was adopted to guarantee string stability of

Manuscript received June 12, 2017; revised October 25, 2017; accepted December 2, 2017.

The authors are with the College of Information Engineering, Zhejiang University of Technology, Hangzhou 310023, China.

De-Feng He is the corresponding author (email: hdfzj@zjut.edu.cn).

This work was supported by the National Natural Science Foundation of China under Grant No. 61773345 and Zhejiang Provincial Natural Science Foundation of China under Grant No.LR17F030004. The authors would like to thank the anonymous reviewers for their helpful comments, which improved the paper.

vehicle platoons in the presence of limited sensory capability of sensors and sensor failure. In those works, the H_∞ controllers were designed by linear matrix inequality technology. In [22], the authors combined the second-order sliding regime and observer approaches to design an ACC tracking control strategy of passenger vehicles with lack of state measurement. Taking static and dynamic measurement errors of vehicles into account, the authors in [23] established bidirectional string stability in order to eliminate the scalability difficulties of vehicle platoons.

Most of the work in the prior literature has guaranteed string stability of vehicle platoons with actuator time-delay, limited sensory capability, and sensor failure. With advances in electronic systems, more safety indexes and physical constraints, such as ride comfort, fuel economy and velocity limit, have been imposed on vehicle platoon controllers in order to improve the driving performance of vehicle platoon systems. To exploit this potential, the model predictive control (MPC) strategy ;is an effective way to solve multi-objective ACC problems in vehicles, particularly in dealing with the constraints of ACC systems [24,25]. The main merits of the MPC-based ACC are that the multi-objective ACC problem is formulated in a unified single objective optimization framework, where the constraints of ACC are systematically satisfied by embedding the optimization problem. Recently several distributed MPC algorithms have been proposed to control vehicle platoons subject to state and input constraints and string stability of the platoon has been established in the face of constraints [26–29]. In general, most of the algorithms were formulated for vehicle platoons without considering any time-delay of communication and actuators. To the best of our knowledge, moreover, the available MPC results of vehicle platoons do not involve on-board sensor concerns, such as limited measuring range, measuring time-dela,y and measurement error. In practical implementation, these are rigid assumptions and deteriorate the driving performances of the platoon, for example, constraint violation, ride dizziness, string instability, and so on. In addition, the optimization problem of MPC presented in most prior studies is computational intensive for the processors currently available in vehicles, which implies that the real-time implementation of MPC may be difficult for vehicle platoons.

In this work, we develop a time-delay feedback MPC strategy for vehicle platoons subject to constraints on limited measurement range and driving behaviors of velocity and acceleration in the presence of time delays. This paper also focuses on the multiple driving performances of vehicle platoons resulting from the consideration of tracking, ride comfort, and fuel economy. They are weighted into a single objective optimization control problem for vehicle platoons. The longitudinal time-delay dynamics of vehicle platoons is used as the predictive model of MPC, where the input increment is used as the input variable of the platoon system. Then a set of MPC controllers is constructed according to the effects of limited measurement range on control of platoons. The controllers are parameterized offline by some free coefficients in order to reduce the computational burden of the online solution of the constrained optimization problem of MPC. The coefficients are online determined by minimizing the objective function of the platoon in a receding horizon fashion. By terminal equality, the zero steady-state spacing error and string stability of the platoon are established to improve the tracking and ride comfort performance of the vehicles. Two typical road scenarios are used to demonstrate the effectiveness of the MPC results presented here.

The remainder of the paper is organized as follows. The vehicle platoon dynamics is modeled in Section II and the corresponding MPC algorithm and the stability analysis are given in Section III. Two classical scenarios are used to verify the proposed method in Section IV. Finally, Section V concludes the paper.

II. PROBLEM FORMULATION

We focus on a simplified platoon of $n>1$ vehicles traveling in a straight line as shown in Fig. 1. The position, velocity and acceleration of the ith (i=0,…,$n-1$) vehicle are denoted by z_i, v_i and a_i, respectively. Let i=0 be the leading vehicle. The velocity and acceleration of the leading vehicle are transmitted periodically to the following vehicles by the vehicular communication network. The velocity, location, and acceleration of the adjacent vehicles are collected by an on-board system, such as radar sensor, camera, and so

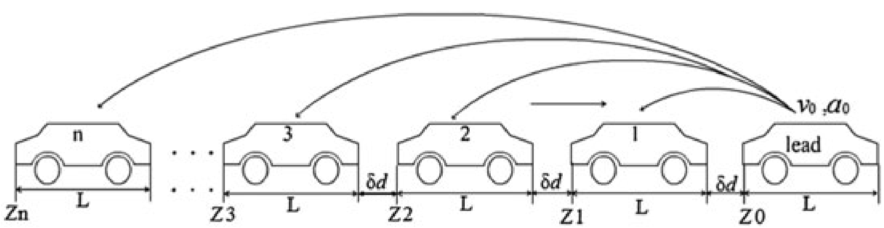


Fig. 1. Platoon of the n vehicles.

on. Moreover, there are some time delays in the actuators of vehicles (e.g., pedal and braking). The desired interval and the length of each vehicle are denoted by constants δ_d and L_i, respectively.

Define the distance error between two vehicles as $q_i = z_{i-1} - z_i - L_i - \delta_d$. The velocity and acceleration errors of the adjacent vehicles are written as

$$\dot{q}_i(t) = v_{i-1}(t) - v_i(t), \ddot{q}_i(t) = a_{i-1}(t) - a_i(t) \quad (1)$$

The acceleration derivative is written as from [25]

$$\dot{a}_i(t) = -\eta_i^{-1} a_i(t) + \eta_i^{-1} u_i(t-\tau) \quad (2)$$

which yields

$$\begin{aligned}\dddot{q}_i(t) &= \dot{a}_{i-1}(t) - \dot{a}_i(t) \\ &= -\eta_i^{-1}\ddot{q}_i(t) + \eta_i^{-1}\Delta u_i(t-\tau)\end{aligned} \quad (3)$$

where η_i is the lag time in tracking the desired acceleration command, τ is the actuator/sensing time delay, u_i is the command acceleration of the i^{th} vehicle and $\Delta u_i = u_i - u_{i-1}$ is the difference of acceleration of the adjacent vehicles for $1 \le i \le n-1$. Note that Δu_i will be selected as the control variable of the ACC state-space model (see Section III) as the state variables are defined as the differences of position, speed and acceleration of between adjacent vehicles.

In the platoon system considered here, the vehicles are subject to the following constraints:

$$\begin{aligned} &q_{\min} \le q_i \le q_{\max} && \Delta v_{\min} \le \dot{q}_i \le \Delta v_{\max} \\ &\Delta a_{\min} \le \ddot{q}_i \le \Delta a_{\max} && \Delta u_{\min} \le \Delta u_i \le \Delta u_{\max} \end{aligned} \quad (4)$$

where constants $q_{\min}<0$, $\Delta v_{\min}<0$, $\Delta a_{\min}<0$ and $\Delta u_{\min}<0$ are the minimum errors of the spacing, speed and acceleration, and the control increment, respectively and constants $q_{\max}>0$, $\Delta v_{\max}>0$, $\Delta a_{\max}>0$ and $\Delta u_{\max}>0$ are the maximum errors of the spacing, speed and acceleration, and the control increment, respectively. The constraints in (4) are determined by the safe driving and ride comfort performances of vehicles. For the platoon system, we select a classic PD type cruise control strategy with the time-delay τ

$$\begin{aligned} u_i(t-\tau) = {} & k_p^q(q_i(t-\tau)) + k_v^q(\dot{q}_i(t-\tau)) \\ & + k_a^q(\ddot{q}_i(t-\tau)) + k_v^e(v_0(t-\tau) - v_i(t-\tau)) \\ & + k_a^e(a_0(t-\tau) - a_i(t-\tau)) \end{aligned} \quad (5)$$

where coefficients $k_p^q, k_v^q, k_a^q, k_v^e, k_a^e$ are the controller gains. The law (5) is the function on the errors of the distance, speed, and acceleration of the ith pair vehicles and the speed and acceleration of the leader. In order to compute these gains, the following objective function is introduced.

$$\begin{aligned} J_i(t_k) = \int_{t_k}^{t_k+T} \big\{ & l_{q_i} q_i^2(t;t_k) + l_{\dot{q}_i}\dot{q}_i^2(t;t_k) + l_{\ddot{q}_i}\ddot{q}_i^2(t;t_k) \\ & + r_i \Delta u_i^2(t-\tau;t_k) \big\} dt \end{aligned} \quad (6)$$

where the sampling time is $t_k = t_0 + k\varepsilon$ and $\varepsilon>0$ is sampling period, $t_0=0$, $k=0,1,2,....$ The first two items in (6) denotes the tracking performance and the third and fourth items denote the ride comfort and dynamic characteristics of vehicles, respectively. The weights $l_{q_i}, l_{\dot{q}_i}, l_{\ddot{q}_i}, r_i$ are positive numbers and $T>\varepsilon$ is the prediction horizon. Note that in equation (6), the weights affect the preferences of objectives and can be tuned by the heuristic try-and-error method in practice (see, for example, [24]).

In real-time applications, on-board sensors, for example, radar and ultrasonic, have limited perception capability, which will effect the platoon controller (5). We consider the following output characteristic of the distance sensors [19]:

$$S_i(q_i) = \begin{cases} q_i & \text{if} \quad q_i < \underline{d}_i - \delta_d \\ q_i\left(1 - \dfrac{D_i}{2\underline{d}_i}\right) + \dfrac{D_i}{2} & \text{if} \quad \underline{d}_i - \delta_d \le q_i \le \overline{d}_i - \delta_d \\ D_i & \text{if} \quad q_i > \overline{d}_i - \delta_d \end{cases} \quad (7)$$

where $\underline{d}_i$ is the lower measurement boundary, $\overline{d}_i$ is the upper measurement boundary, D_i is a constant distance, and S_i is the actual inter-spacing measured by the sensors. For clarity, let $\underline{d}_i = \underline{d}$, $\overline{d}_i = \overline{d}$, $D_i = D$. Substituting (7) into (5), it is obtained that

$$\Delta u_i(t-\tau) = \begin{cases} k_p^q q_{i-1}(t-\tau) + k_v^q \dot{q}_{i-1}(t-\tau) + k_a^q \ddot{q}_{i-1}(t-\tau) - k_p^q q_i(t-\tau) & \\ -\left(k_v^q + k_v^e\right)\dot{q}_i(t-\tau) - \left(k_a^q + k_a^e\right)\ddot{q}_i(t-\tau), & \text{if } q_i(t-\tau) \le \underline{d} - \delta_d \\ \widetilde{d}\left[k_p^q q_{i-1}(t-\tau) + k_v^q \dot{q}_{i-1}(t-\tau) + k_a^q \ddot{q}_{i-1}(t-\tau)\right] - \widetilde{d} k_p^q q_i(t-\tau) & \\ -\left(\widetilde{d} k_v^q + k_v^e\right)\dot{q}_i(t-\tau) - \left(\widetilde{d} k_a^q + k_a^e\right)\ddot{q}_i(t-\tau), & \text{if } \underline{d} - \delta_d \le q_i(t-\tau) \le \overline{d} - \delta_d \\ -k_v^e \dot{q}_i(t-\tau) - k_a^e \ddot{q}_i(t-\tau), & \text{if } q_i(t-\tau) \ge \overline{d} - \delta_d \end{cases} \quad (8)$$

with the constant $\widetilde{d} = 1 - D/2\underline{d}$. Clearly, the output characteristic of the sensors leads to a switching effect of the platoon controller.

The goal of this paper is to present a time-delay feedback strategy for vehicle platoons subject to constraints on limited measurement range and driving behaviors of velocity and acceleration in the presence of time delays, which guarantees the following properties: 1) the vehicle cruising stability [20], that is, the following vehicles rapidly track the jerk of the leading vehicle and the speed differences tend to zero; 2) string stability [30], that is, the string stability transfer function

$$SS_{\Delta_i}(s) = \Delta_i(s)/\Delta_{i-1}(s), \quad i \geq 1, s \in C \tag{9}$$

satisfies

$$\|SS_{\Delta_i}(j\omega)\|_\infty \leq 1, \quad \forall i \geq 1, \omega \geq 0 \tag{10}$$

where $\|\cdot\|_\infty$ is the ∞-norm, $\Delta_i(s)=L(\delta_i)$, $\delta_i \in R$ is the signal of interest for the evaluation of the string stability, and L denotes the Laplace operator. The inequality (10) describes the maximal amplification of perturbation along the string of vehicles. In this paper we use the MPC method to develop the time-delay feedback control for the vehicle platoon system.

III. PREDICTIVE CONTROL FOR VEHICLE PLATOON

Consider the dynamics (1)-(3) and let $x(k) = Col[x_i(k)]_{i=1}^{n-1}$ and $x_i(k) = [q_i(k)\ \dot{q}_i(k)\ \ddot{q}_i(k)]^T$. We have the discrete-time model with a sampling period $\varepsilon > 0$

$$x(k+1) = Ax(k) + B\Delta u(k-d) \tag{11}$$

where the integer $d=\tau/\varepsilon$ is the time delay in the discrete-time system, $A=\text{diag}\{A_1, A_2, \ldots, A_{n-1}\}$ and $B=\text{diag}\{B_1, B_2, \ldots, B_{n-1}\}$ with $A_i = \begin{bmatrix} 1 & \varepsilon & 0 \\ 0 & 1 & \varepsilon \\ 0 & 0 & 1-\varepsilon/\eta_i \end{bmatrix}$ and $B_i=[0\ \ 0\ \ -\varepsilon/\eta_i]^T$ for $i=1,\ldots,n-1$. Note that here the time delay is assumed to be known as there generally exists a range of 0.5~2.0 sec in practice. The constraints in (4) can be expressed as $x_{\min} \leq x(k) \leq x_{\max}$ and $\Delta u_{\min} \leq u(k) \leq \Delta u_{\max}$, where $x_{\min}=[q_{\min}, \Delta v_{\min}, \Delta a_{\min}]^T$ and $x_{\max}=[q_{\max}, \Delta v_{\max}, \Delta a_{\max}]^T$. Moreover, the state feedback control law is re-written as

$$\Delta u(k) = Kx(k) \tag{12}$$

with $K = \begin{bmatrix} K_1^1 & 0 & \cdots & 0 \\ K_2^2 & K_2^1 & \cdots & 0 \\ 0 & \cdots & \ddots & \cdots \\ 0 & \cdots & K_{n-1}^2 & K_{n-1}^1 \end{bmatrix}$. From (8), the components in K are defined as follows:

$K_i^1 = \begin{bmatrix} k_p^q & k_v^q + k_v^e & k_a^q + k_a^e \end{bmatrix}$ and $K_i^2 = \begin{bmatrix} -k_p^q & -k_v^q & -k_a^q \end{bmatrix}$ if $q_i \leq \underline{d} - \delta_d$, $K_i^1 = \begin{bmatrix} \widetilde{d}k_p^q & \widetilde{d}k_v^q + k_v^e & \widetilde{d}k_a^q + k_a^e \end{bmatrix}$ and $K_i^2 = \begin{bmatrix} -\widetilde{d}k_p^q & -\widetilde{d}k_v^q & -\widetilde{d}k_a^q \end{bmatrix}$ if $\underline{d} - \delta_d \leq q_i \leq \overline{d} - \delta_d$, and $K_i^1 = \begin{bmatrix} 0 & k_v^e & k_a^e \end{bmatrix}$ and $K_i^2 = [0\ \ 0\ 0]$ if $q_i \geq \overline{d} - \delta_d$.

Due to the time-delay d of signal propagation, the vehicle platoon system has a range relation with the different values of prediction horizon p. We consider the prediction states of the vehicle platoon system, which are evaluated in different prediction horizons.

For $1 \leq p \leq d+1$, according to (11) and (12) we have

$$\begin{aligned} x(k+1|k) &= Ax(k|k) + BKx(k-d|k) \\ x(k+2|k) &= A^2x(k|k) + ABKx(k-d|k) + BKx(k+1-d|k) \\ &\vdots \\ x(k+d|k) &= A^dx(k|k) + A^{d-1}BKx(k-d|k) + \cdots + BKx(k-1|k) \\ x(k+d+1|k) &= \left(A^{d+1} + BK\right)x(k|k) + A^dBKx(k-d|k) + \cdots ABKx(k-1|k) \end{aligned}$$

Note that at the current time k, the states $x(k\text{-}d/k)$, $x(k\text{+}1\text{-}d/k)$, …, $x(k\text{-}1|k)$ are the history state information, and $x(k|k)=x(k)$ is the current state. Let $X(k)=[x(k\text{-}d/k)^T\ x(k\text{+}1\text{-}d/k)^T\ \ldots\ x(k\text{-}1|k)^T\ x(k/k)^T]^T$. The above predictions can be re-written as

$$\begin{bmatrix} x(k+1|k) \\ x(k+2|k) \\ \vdots \\ x(k+d|k) \\ x(k+d+1|k) \end{bmatrix} = \begin{bmatrix} BK & 0 & \cdots & 0 & A \\ ABK & BK & \cdots & 0 & A^2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ A^{d-1}BK & A^{d-2}BK & \cdots & BK & A^d \\ A^dBK & A^{d-1}BK & \cdots & ABK & A^{d+1} + BK \end{bmatrix} X(k) \tag{13}$$

Moreover, a general formula for different prediction time lengths $(m\text{-}1)d+m \leq p \leq m(d+1)$, $m=1,2,..$ is deduced as

$$\begin{bmatrix} x(k+(m-1)d+m|k) \\ x(k+(m-1)d+m+1|k) \\ \vdots \\ x(k+md+m-1|k) \\ x(k+md+m|k) \end{bmatrix} = \begin{bmatrix} BK & 0 & \cdots & 0 & A \\ ABK & BK & \cdots & 0 & A^2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ A^{d-1}BK & A^{d-2}BK & \cdots & BK & A^d \\ A^dBK & A^{d-1}BK & \cdots & ABK & A^{d+1}+BK \end{bmatrix}^m X(k) \tag{14}$$

Now the finite horizon optimal control problem of the vehicle platoon is defined as

$$\begin{aligned} &\min_K \left\{ J(x(k)) = \sum_{i=0}^{p-1} \left[\|x(k+i|k)\|_Q^2 + \|\Delta u(k+i-d|k)\|_R^2 \right] \right\} \\ &s.t.\ \ x(k+i+1|k) = Ax(k+i|k) + B\Delta u(k+i-d|k) \\ &\quad \Delta u_{\min} \le \Delta u(k+i-d|k) \le \Delta u_{\max} \\ &\quad x_{\min} \le x(k+i|k) \le x_{\max} \\ &\quad \Delta u(k+i-d|k) = Kx(k+i-d|k), i = 0,1,\cdots,p-1 \\ &\quad x(k+i-d|k) = x(k+i-d), \qquad i = 0,1,\cdots,p-1 \\ &\quad x(k|k) = x(k),\ \ x(k+p|k) = 0 \end{aligned} \tag{15}$$

$$\begin{aligned} J_{k+1} &= \sum_{i=0}^{p-1} \left(\|x(k+1+i|k+1)\|_Q^2 + \|\Delta\hat{u}(k+1+i-d|k+1)\|_R^2 \right) \\ &= \sum_{i=1}^{p-1} \left(\|x(k+i|k)\|_Q^2 + \|\Delta u^*(k+i-d|k)\|_R^2 \right) \\ &= \sum_{i=0}^{p-1} \left(\|x(k+i|k)\|_Q^2 + \|\Delta u^*(k+i-d|k)\|_R^2 \right) - \|x(k|k)\|_Q^2 - \|\Delta u^*(k-d|k)\|_R^2 \\ &= J_k^* - \|x(k|k)\|_Q^2 - \|\Delta u^*(k-d|k)\|_R^2 \\ &\le J_k^* \end{aligned}$$

where $x(k)$ is the state measurement at time k=0,1,… the terminal equality $x(k+p|k)=0$, the weighted diagonal matrices $Q=\text{diag}\{Q_1, Q_2, \ldots, Q_{n-1}\}$, $Q_i=\text{diag}\{q_1, q_2, q_3\}>0$, and $R=\text{diag}\{r_1, r_2, \ldots, r_{n-1}\}>0$. Note that due to the delay d, the terminal constraint is used to force the terminal states of the platoon back to the equilibrium point.

Theorem 1. Assume that the platoon is zero state observable and the optimization problem (15) has the solution at initial time k=0. Then the problem (15) has recursive feasibility and the system (11) in closed-loop with (12) with the optimal solution K^* has asymptotic stability.

Proof. At the current time k, it is assumed that there exists an optimal solution of (15), K^*, to ensure the control law (12) satisfies the constraints of the vehicle platoon system. At the next time k+1, construct the gain $\hat{K}(k+1) = K^*(k)$ and the corresponding control law is expressed as

$$\Delta\hat{u}(k+1) = \begin{cases} K^*(k)x(k+1+i-d|k+1), & i = 0,1,\ldots p+d-1 \\ 0, & i \ge p+d \end{cases} \tag{16}$$

By the nature of $K^*(k)$ and the terminal equality constraint, the control law (16) satisfies the constraints of (15) at time k+1. Hence, the gain is a feasible solution of (15). Namely, the problem (15) has recursive feasibility. Because (15) has a solution at the initial time k=0, the problem is solvable at any time k.

Let $K^*(k)$ and $K^*(k+1)$ be the optimal solutions at k and k+1, respectively. The corresponding control laws are denoted as $\Delta u^*(k)$ and $\Delta u^*(k+1)$. Substituting them into $J(x)$ and making differential calculation, we have

Therefore J_k has an upper boundary J_0^* and $J_{k+1}^* \le J_{k+1} \le J_k^*$. That is, J_k^* is a monotonically decreasing function. By Lyapunov's argument, the closed-loop system of vehicle platoon is asymptotically stable.

Theorem 2. Consider the string stability transfer function (9). Under the assumptions in Theorem 1, the inequalities

$|q_i(\mathrm{j}\omega)/q_{i-1}(\mathrm{j}\omega)|\le 1$ hold for any i=1,…,n and $\omega>0$ if the following conditions are satisfied.

$$
\begin{aligned}
&(\mathrm{a})\left[2\left(\tilde{d}k_a^q+k_a^e\right)+2\tilde{d}k_a^qk_a^e+\left(k_a^e\right)^2-2\eta_i\tau\tilde{d}k_p^q-2\left(\tilde{d}k_v^q+k_v^e\right)(\eta_i-\tau)\right]\ge 0\\
&(\mathrm{b})\left[\left(k_v^e\right)^2-2\left(\tilde{d}k_p^q+\tilde{d}k_p^qk_a^e\right)+2\tilde{d}k_v^qk_v^e\right]\ge 0\\
&(\mathrm{c})\eta_i\tau\left(1+\tilde{d}k_a^q+k_a^e\right)\ge 0
\end{aligned}
\tag{17}
$$

Proof. In order to establish string stability of the vehicle platoon, substituting (8) into (3) we obtain that

$$
\dot{a}_{i-1}(t)-\dot{a}_i(t)=\begin{cases}
-\frac{1}{\eta_i}\ddot{q}(t)+\frac{1}{\eta_i}\left[k_p^q(q_{i-1}(t)-q_i(t))+k_v^q(\dot{q}_{i-1}(t)-\dot{q}_i(t))+k_a^q(\ddot{q}_{i-1}(t)-\ddot{q}_i(t))+k_v^e(v_{i-1}(t)-v_i(t))+k_a^e(a_{i-1}(t)-a_i(t))\right] & \text{if } q_i\le \underline{d}-\delta_d\\
-\frac{1}{\eta_i}\ddot{q}(t)+\frac{1}{\eta_i}\left[k_p^q\tilde{d}(q_{i-1}(t)-q_i(t))+k_v^q\tilde{d}(\dot{q}_{i-1}(t)-\dot{q}_i(t))+k_a^q\tilde{d}(\ddot{q}_{i-1}(t)-\ddot{q}_i(t))+k_v^e(v_{i-1}(t)-v_i(t))+k_a^e(a_{i-1}(t)-a_i(t))\right] & \text{if } \underline{d}-\delta_d<q_i<\overline{d}-\delta_d\\
-\frac{1}{\eta_i}\ddot{q}_i(t)+\frac{1}{\eta_i}\left[k_v^e(v_{i-1}(t)-v_i(t))+k_a^e(a_{i-1}(t)-a_i(t))\right] & \text{if } q_i\ge \overline{d}-\delta_d
\end{cases}
\tag{18}
$$

Taking Laplace transforms of (18) leads to

$$
G(s)=\frac{q_i(s)}{q_{i-1}(s)}=\begin{cases}
\dfrac{\tilde{d}k_p^q+\tilde{d}k_v^qs+\tilde{d}k_a^qs^2}{(\eta_is^3+s^2)e^{\tau s}+\tilde{d}k_p^q+\left(\tilde{d}k_v^e+k_v^q\right)s+\left(\tilde{d}k_a^q+k_a^e\right)s^2} & q_i<\overline{d}-\delta_d\\
1 & q_i\ge\overline{d}-\delta_d
\end{cases}
\tag{19}
$$

Let $e^{\tau s}$=1-τs. Substituting (19) into the frequency domain yields

$$
G(j\omega)=\frac{q_i(j\omega)}{q_{i-1}(j\omega)}=\begin{cases}
\dfrac{\tilde{d}k_p^q+\tilde{d}k_v^q\omega j-\tilde{d}k_a^q\omega^2}{(-\eta_i\omega^3j-\omega^2)(1-\tau\omega j)+\tilde{d}k_p^q+\left(\tilde{d}k_v^e+k_v^q\right)\omega j-\left(\tilde{d}k_a^q+k_a^e\right)\omega^2} & q_i<\overline{d}-\delta_d\\
1 & q_i\ge\overline{d}-\delta_d
\end{cases}
\tag{20}
$$

Then

$$
\|G(j\omega)\|=\left\|\frac{q_i(j\omega)}{q_{i-1}(j\omega)}\right\|=\sqrt{\frac{a}{a+b}}\le 1 \tag{21}
$$

where

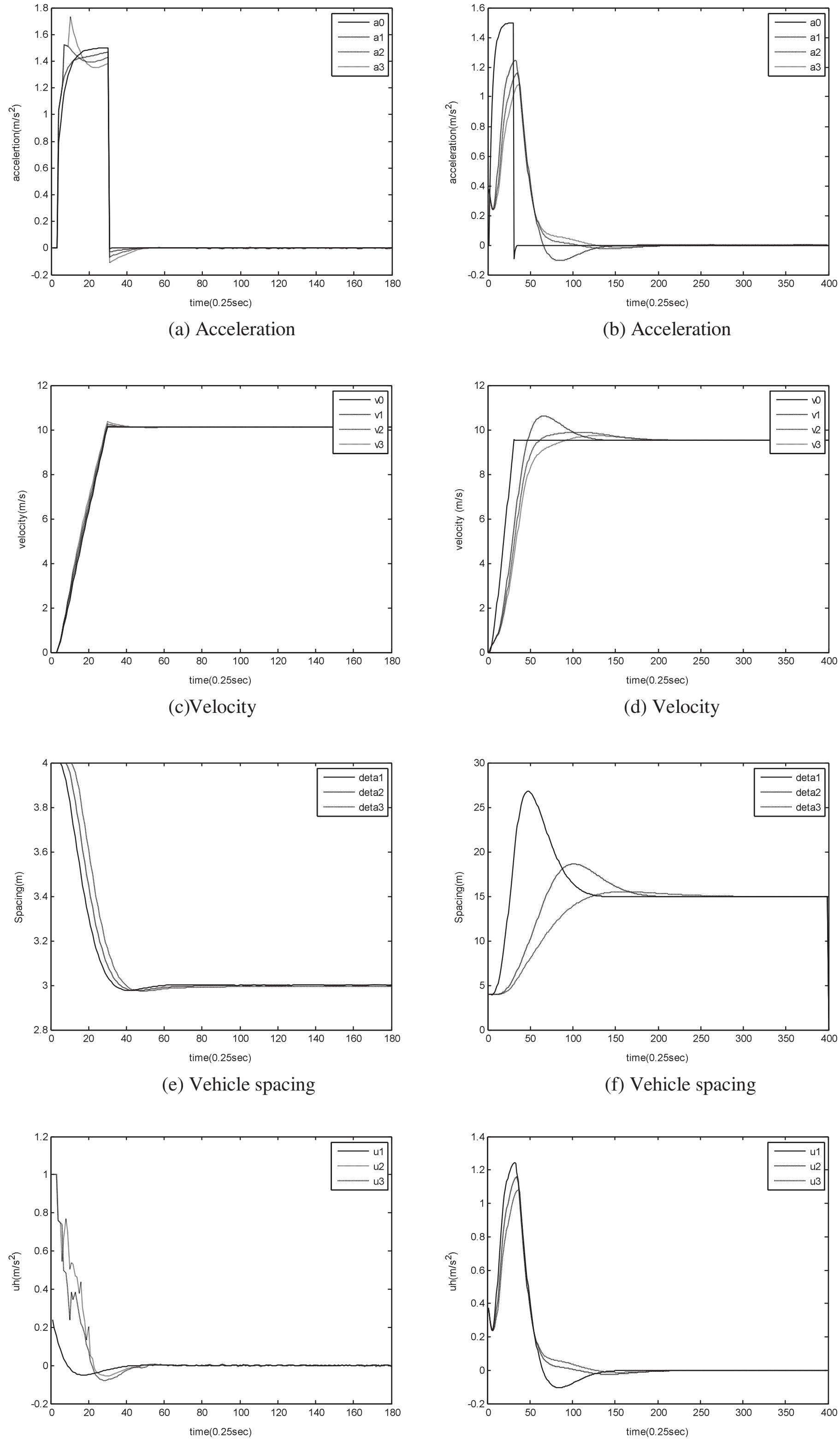
$$
\begin{aligned}
a&=\tilde{d}^2\left(k_p^q\right)^2+\left[\left(\tilde{d}^2\left(k_v^q\right)^2-2\tilde{d}^2k_a^qk_p^q\right)\omega^2\right]+\tilde{d}^2\left(k_a^q\right)^2\omega^4>0,\\
b&=\eta_i^2\tau^2\omega^8+(\eta_i-\tau)^2\omega^6+\omega^4+[2\left(\tilde{d}k_a^q+k_a^e\right)+2\tilde{d}k_a^qk_a^e+\left(k_a^e\right)^2-2\eta_i\tau\tilde{d}k_p^q\\
&\quad-2\left(\tilde{d}k_v^q+k_v^e\right)(\eta_i-\tau)]\omega^4+\left[\left(k_v^e\right)^2-2\left(\tilde{d}k_p^q+\tilde{d}k_p^qk_a^e\right)+2\tilde{d}k_v^qk_v^e\right]\omega^2+2\eta_i\tau\left(1+\tilde{d}k_a^q+k_a^e\right)\omega^6
\end{aligned}
$$

Since $(\eta_i)^2\tau^2\omega^8\ge 0$, $(\eta_i$-$\tau)^2\omega^6\ge 0$, $\omega^4\ge 0$, we have the following inequality:

$$
\begin{aligned}
b\ge&\left[2\left(\tilde{d}k_a^q+k_a^e\right)+2\tilde{d}k_a^qk_a^e+\left(k_a^e\right)^2-2\eta_i\tau\tilde{d}k_p^q-2\left(\tilde{d}k_v^q+k_v^e\right)(\eta_i-\tau)\right]\omega^4\\
&+\left[\left(k_v^e\right)^2-2\left(\tilde{d}k_p^q+\tilde{d}k_p^qk_a^e\right)+2\tilde{d}k_v^qk_v^e\right]\omega^2+2\eta_i\tau\left(1+\tilde{d}k_a^q+k_a^e\right)\omega^6
\end{aligned}
$$

Clearly, we have $b\ge 0$ and consequently $\|G(j\omega)\|\le 1$ if the condition (17) holds. This completes the proof.

Remark 1. The string stability property guarantees the whole vehicle platoon system with no collision and dropping out. It should be pointed out that the conditions in (17) are sufficient to establish string stability of vehicle platoons. In other words, string stability may be



(a) Acceleration

(b) Acceleration

(c)Velocity

(d) Velocity

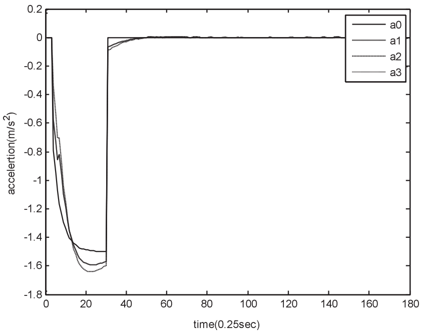
(e) Vehicle spacing

(f) Vehicle spacing

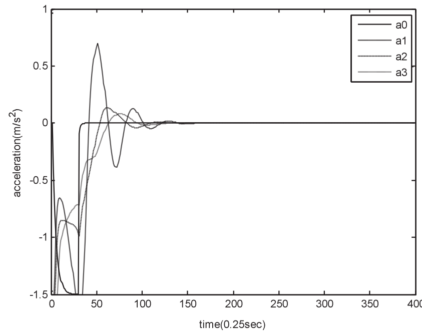
(g) Control input

(h) Control input

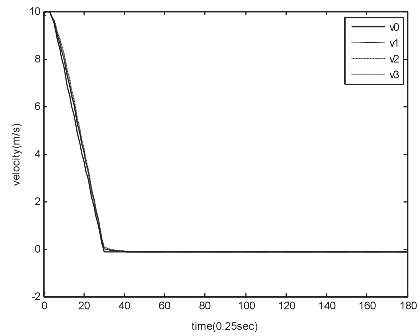
Fig. 2. Responses (a), (c), (e), and (g) are obtained by our method and (b), (d), (f), and (h) are obtained by the IDM method.



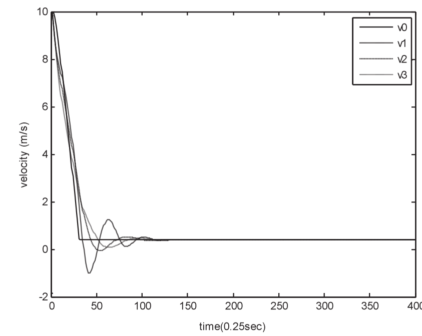
(a) Acceleration



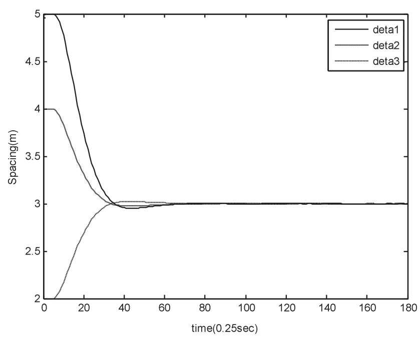
(b) Acceleration



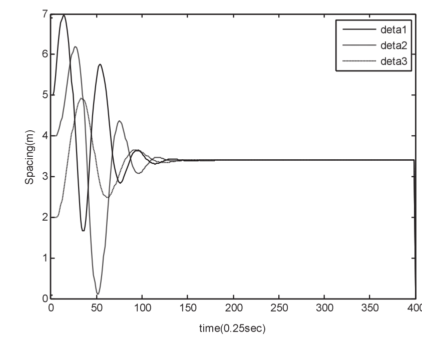
(c) Velocity



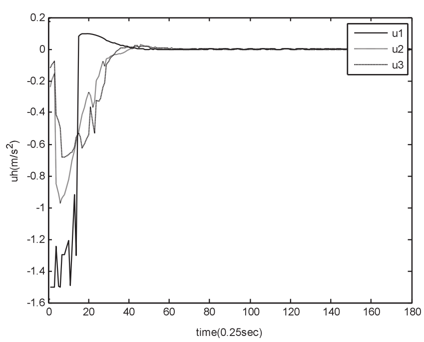
(d) Velocity



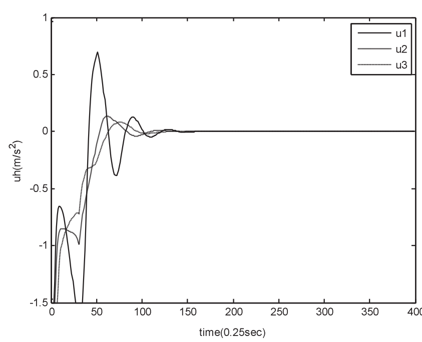
(e) Vehicle spacing



(f) Vehicle spacing



(g) Control input



(h) Control input

Fig. 3. Responses (a), (c), (e), and (g) are obtained by our method and (b), (d), (f), and (h) are obtained by the IDM method.

guaranteed for a vehicle platoon even when the conditions are not satisfied.

IV. SIMULATION EXAMPLES

We compare the proposed method with an Intelligent Driver Model (IDM) [31] to illustrate the effectiveness of the presented results. According to the different range of sensors, the initialization value of the gain is as $g(j,k)$=1 with j=1,2,3,…,15 and the lower and upper boundaries of sensors are $\underline{d} = 4$and $\overline{d} = 8$, respectively. The distance constant D=6, the lag time in tracking the desired acceleration command η_i=0.25 sec, the expected spacing δ_d=3 m, $q_{\min}=-2$, $q_{\max}=2$, $\Delta v_{\min}=-3$, $\Delta v_{\max}=3$, $a_{\min}=-1.5$, $a_{\max}=1.5$, $\Delta u_{\min}=-1.5$, $\Delta u_{\max}=1.5$, and the length of vehicles L_i=4. The time delay τ=2.0 sec and the sampling time is 0.25 sec.

4.1 Scenario 1: Start and acceleration of the leading vehicle

The initial simulation states of the four vehicles are set as [40,0,0], [32,0,0], [24,0,0], and [16,0,0], respectively. Namely, the initial vehicle spacing is more than the desired spacing. Fig. 2 shows the results obtained by the proposed method and the IDM, respectively.

It is observed from Fig. 2 that both methods can stabilize the platoon whilst the time to reach the steady state is 160s for the method proposed here versus 300s for IDM. From Fig. 2 (a)–(b), the largest change rate of the acceleration between adjacent vehicles for IDM is two times that for the proposed method in this paper. Note that the smaller the change rate of the acceleration, the more comfortable the passengers feel. Hence, the proposed method can improve the ride comfort of vehicles. Again, one can see from Fig. 2 (f) that the IDM has almost four times the spacing of that for the proposed method here, which can cause the vehicle queue jumping phenomenon. The reason for this phenomenon is mainly that, by the IDM, the command acceleration is computed by the current state information and is assumed to be able to immediately react to any behavior change in the vehicles. However, in practice a certain reaction time exists in which to respond to the change in traffic conditions. As a result, the following vehicles with IMD cannot track the leading vehicle efficiently and therefore do not have good performance. On the other hand, one can see from Fig. 2 (c) and (e) that the proposed method can track the leading vehicle rapidly and steady-state spacing reaches the desired spacing of 3 m. These improvements can increase the capacity of roads significantly. Note that the veracity of conditions in Theorem 2 can be directly examined by computation at each sampling time, which is omitted here due to space limitations.

4.2 Scenario 2: Emergency braking in the leading vehicle

The initial states of the four vehicles are set as [40,10,0], [31,10,0], [23,10,0], and [17,10,0], respectively. Initialization means that at a certain moment, the velocity of each vehicle is the same but the spacing between the adjacent vehicles is different.

Fig. 3 pictures the simulation results obtained by the proposed method and the IDM, respectively. It is seen that as for scenario 1, both methods can stabilize the platoon system although the adjacent spacing differs. Comparing Fig. 3 (a) and (b), one can see that the change rates of acceleration obtained by the IDM typically have larger fluctuation than those by the proposed method. As a result, passengers may feel uncomfortable and fuel consumption will be increased. Moreover, from Fig. 3 (d) we can know that the steady-state velocity by the IDM is less than zero, which clearly does not accord with the practical situation and will result in some accidents. The main reasons causing these is that the IDM has no ability to predict the behaviors of surrounding vehicles and to cope with time delays from actuators. Finally, it is observed from Fig. 3 (e) and (f) that the desired spacing can be achieved by the proposed method but not for the IDM. Similarly, this will cause some traffic accidents by the IDM.

V. CONCLUSIONS

In this paper, an MPC-based method is proposed to control vehicle platoons considering sensors with limited measurement range and time-delay of actuators. The string stability of the vehicle platoon is established where certain condition create constraints. Simulation examples show that the MPC method can better track the acceleration and speed of the leading vehicle, compared with the available IDM method. In future research, vehicular network features and varying delay of vehicles may be pursued from the viewpoints of consensus of multi-agent systems [8,32].

REFERENCES

1. Tai, M. H., P. Hingwe, and M. Tomizuka, "Modeling and control of steering system of heavy vehicles for automated highway systems," *IEEE Trans. Mechatronics*, Vol. 9, pp. 609–618 (2004).
2. van Arem, B., C. J. G. van Driel, and R. Visser, "The impact of cooperative adaptive cruise control on

traffic-flow characteristics," *IEEE Trans. Intell. Transp. Syst.*, Vol. 7, pp. 429–436 (2007).
3. Kavathekar, P. and Y. Q. Chen, "Vehicle platooning: A brief survey and categorization," 'Proc. ASME IDETC/CIE. Washington D. C.,. Vol. 3, pp. 829–845 (2012)'
4. Ioannou, P., "Guest editorial adaptive cruise control systems special issue," *IEEE Trans. Intell. Transp. Syst.*, Vol. 4, pp. 113–114 (2003).
5. Milanés, V., and S. E. Shladover, "Modeling cooperative and autonomous adaptive cruise control dynamic responses using experimental data," '*Transport. Res. C-Emer.*' Vol. 48, pp. 285–300 (2014).
6. Ploeg, J., D. P. Shukla, N. van de Wouw, *et al*., "Controller synthesis for string stability of vehicle platoons," *IEEE Trans. Intell. Transp. Syst.*, Vol. 15, No. 2, pp. 854–865 (2014).
7. Oncu, S., J. Ploeg, N. van de Wouw, *et al*., "Cooperative adaptive cruise control: Network-aware analysis of string stability," *IEEE Trans. Intell. Transp. Syst.*, Vol. 15, No. 4, pp. 1527–1537 (2014).
8. Zou, Y. Y., J. Lam, Y. G. Niu, *et al*., "Constrained predictive control synthesis for quantized systems with Markovian data loss," *Automatica*, Vol. 55, pp. 217–225 (2015).
9. Bernardo, M., A. Salvi, and S. Santini, "Distributed consensus strategy for platooning of vehicles in the presence of time-varying heterogeneous communication delays," *IEEE Trans. Intell. Transp. Syst.*, Vol. 16, No. 1, pp. 102–111 (2015).
10. Ge, J. I., and G. Orosz, "Dynamics of connected vehicle systems with delayed acceleration feedback," *Transport. Res. C-Emer.*, Vol. 46, pp. 46–64 (2014).
11. Liu, X. H., A. Goldsmith, S. S. Mahal, et al., "Effects of communication delay on string stability in vehicle platoons," *Proc. IEEE Intl. Conf. Intell. Transport Syst*., pp. 625-630 (2001).
12. Xiao, L. Y., S. Darbha, and F. Gao, "Stability of string of adaptive cruise control vehicles with parasitic delays and lags," *Proc. 11th Int. IEEE Conf. Intell. Transportation Syst*. Beijing China, pp. 1101-1106 (2008).
13. Naus, G. J. L., R. P. A. Vugts, J. Ploeg, *et al*., "String-stable CACC design and experimental validation: A frequency-domain approach," *IEEE Trans. Veh. Technol.*, Vol. 59, pp. 4268–4279 (2010).
14. Xiao, L. Y., and F. Gao, "Practical string stability of platoon of adaptive cruise control vehicles," *IEEE Trans. Intell. Transp. Syst.*, Vol. 12, pp. 1184–1194 (2011).
15. Yanakiev, D., and I. Kanellakopoulos, "Longitudinal control of automated CHVs with significant actuator delays," *IEEE Trans. Veh. Technol.*, Vol. 50, pp. 1289–1297 (2001).
16. Desjardins, C., and B. Chaib-draa, "Cooperative adaptive cruise control: A reinforcement learning approach," *IEEE Trans. Intell. Transp. Syst.*, Vol. 12, No. 4, pp. 1248–1260 (2011).
17. Jia, Z., A. Balasuriya, and S. Challa, "Sensor fusion-based visual target tracking for autonomous vehicles with the out-of-sequence measurement solution," *Robot. Auton. Syst.*, Vol. 56, No. 2, pp. 157–176 (2008).
18. Middleton, R. H., and J. H. Braslavsky, "String instability in classes of linear time invariant formation control with limited communication range," *IEEE Trans. Autom. Control*, Vol. 55, No. 7, pp. 1519–1530 (2010).
19. Guo, G., and W. Yue, "Autonomous platoon control allowing range-limited sensors," *IEEE Trans. Veh. Technol.*, Vol. 61, pp. 2901–2912 (2012).
20. Wang, Q., G. Guo, and B. Cai, "Distributed receding horizon control for fuel-efficient and safe vehicle platooning," *Sci. China-Technol. Sci.*, Vol. 59, No. 12, pp. 1953–1962 (2016).
21. Guo, G., and W. Yue, "Sampled-data cooperative adaptive cruise control of vehicles with sensor failures," *IEEE Trans. Intell. Transp. Syst.*, Vol. 15, pp. 2404–2418 (2014).
22. Ferrara, A., and P. Pisu, "Minimum sensor second-order sliding mode longitudinal control of passenger vehicles," *IEEE Trans. Intell. Transp. Syst.*, Vol. 5, pp. 20–32 (2004).
23. Knorn, S., A. Donaire, J. C. Aguero, *et al*., "Scalability of bidirectional vehicle strings with static and dynamic measurement errors," *Automatica*, Vol. 62, pp. 208–212 (2015).
24. Li, S., Z. Jia, K. Li, and B. Cheng, "Fast online computation of a model predictive controller and its application to fuel economy-oriented adaptive cruise control," *IEEE Trans. Intell. Transp. Syst.*, Vol. 16, No. 3, pp. 1199–1209 (2015).
25. Zhao, R. C., P. K. Wong, Z. C. Xie, *et al*., "Real-time weighted multi-objective model predictive controller for adaptive cruise control systems," *Int. J. Automot. Technol.*, Vol. 18, No. 2, pp. 279–292 (2017).
26. Kianfar, R., P. Falcone, and J. Fredriksson, "A distributed model predictive control approach to active steering control of string stable cooperative vehicle platoon," *7th IFAC Symp. Adv. Automotive Control,* Madrid, Spain, pp. 750-755 (2013).
27. Li, H. P., Y. Shi, and W. S. Yan, "Distributed receding horizon control of constrained nonlinear vehicle formation with guaranteed γ-gain stability," *Automatica*, Vol. 68, pp. 148–154 (2016).
28. Dunbar, W. B., and D. S. Caveney, "Distributed receding horizon control of vehicle platoons: Stability and

string stability," *IEEE Trans. Autom. Control*, Vol. 57, No. 3, pp. 620–633 (2012).
29. Zhang, H. T., Z. M. Cheng, and G. Chen, "Model predictive flocking control for second-order multi-agent systems with input constraints," *IEEE Trans. Circuits Syst. I-Regul. Pap.*, Vol. 62, No. 6, pp. 1599–1606 (2015).
30. Swaroop, D., and J. K. Hedrick, "String stability of interconnected systems," *IEEE Trans. Autom. Control*, Vol. 41, No. 3, pp. 349–357 (1996).
31. Kesting, A., M. Treiber, and D. Helbing, "Enhanced intelligent driver model to access the impact of driving strategies on traffic capacity," *Philos. T. R. Soc. A*, Vol. 368, pp. 4585–4605 (2010).
32. Zhang, H. T., B. Liu, Z. M. Cheng, and G. Chen, "Model predictive flocking control of the Cucker-Smale multi-agent model with input constraints," *IEEE Trans. Circuits Syst. I-Regul. Pap.*, Vol. 63, No. 8, pp. 1265–1275 (2016).

Shi-Ming Yu received his PhD from the Department of Automation at Zhejiang University in 2001. He is currently a professor at the College of Information Engineering, Zhejiang University of Technology. His research interests include model predictive control and networked control systems.

Sai-Nan Wu received her bachelor degree from Jiangsu University of Science and Technology in 2015. She will receive her masters degree from the College of Information Engineering, Zhejiang University of Technology in June 2018. Her research interests include model predictive control of vehicle platoons.

Yun-Bo Zhao received his PhD in control engineering from the University of South Wales, UK in 2008. He is currently a professor at the College of Information Engineering, Zhejiang University of Technology. His research interests include networked control systems and systems biology.

De-Feng He received his PhD from the Department of Automation in University of Science and Technology of China in 2008. He is currently a professor at the College of Information Engineering, Zhejiang University of Technology. His research interests include model predictive control theory and applications.