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Research Article

Event-Triggered Bipartite Consensus of Single-Integrator Multi-Agent Systems with Measurement Noise

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Event-triggered bipartite consensus of single-integrator multi-agent systems is investigated in the presence of measurement noise. A time-varying gain function is proposed in the event-triggered bipartite consensus protocol to reduce the negative effects of the noise corrupted information processed by the agents. Using the state transition matrix, Itô formula, and the algebraic graph theory, necessary and sufficient conditions are given for the proposed protocol to yield mean square bipartite consensus. We find that the weakest communication requirement to ensure the mean square bipartite consensus under event-triggered protocol is that the signed digraph is structurally balanced and contains a spanning tree. Numerical examples validated the theoretical findings where the system shows no Zeno behavior.

1. Introduction

Recent years have witnessed the great achievements in studying the consensus problem of multi-agent systems (MASs) which has broad applications in various fields [1–8]. We notice that in these mentioned works interactions among agents are all assumed to be cooperative to achieve consensus. However, it is very natural to see, in many real examples, that in MASs some agents cooperate while others compete, and MASs with competitive interactions can introduce more complex behaviors. To quantitatively model such a scenario, the concept of bipartite consensus, i.e., agents agree on a certain quantity with the equal modulus but different signs, has been proposed [9], and many achievements have been made [9–18]. In [9], for single-integrator MASs, a linear feedback protocol is designed and under the assumption that the communication topology $\mathcal{G}$ is strongly connected, the MAS is proved to achieve bipartite consensus if and only if $\mathcal{G}$ is structurally balanced. Then, in [10], the communication condition in [9] is relaxed to containing a spanning tree. In [11], the communication topology in [9] is extended to the time-varying case.

It is worth noting that the above literatures mainly focus on continuous feedback protocols, where the agent state is monitored continuously and its controller is updated all the time. However, updating the controller in real-time easily increases the computational burden. Therefore, reducing the update frequency for a trade-off between the system performance and the resource usage is usually desired. This requirement then naturally brings event-triggered schemes into consideration, which updates only at some predetermined discrete time instants. Event-triggered techniques have already been widely used in traditional consensus problems of MASs [19–27]. For example, a self-triggered protocol is proposed in [19] and a decentralized event-triggered protocol ensuring average consensus is proposed in [20] for single-integrator MASs, time-dependent triggering functions are investigated in [24] for second-order MASs, and event-triggered consensus problems are considered in [25, 26] for general linear systems, just name a few. Despite these achievements, event-triggered protocols have not been well studied for bipartite consensus [28, 29], which thus motivates the present study.

In another parallel line, measurement noise is unavoidable in practice, making the investigation on the event-triggered bipartite consensus of MASs with noise even interesting. In fact, studies on bipartite consensus with measurement noise can be found in [13, 16–18], which are however all with time-triggered controllers. Event-triggered bipartite consensus for MASs with measurement noise still remains to tackle.

In this paper, we investigate event-triggered bipartite consensus for single-integrator MASs with measurement noise. A time-varying control gain is introduced into the event-triggered protocols, leading to a time-varying closed-loop system. With the help of the state transition matrix and stochastic analysis theory, the closed-loop system is analyzed. Necessary and sufficient conditions for the system to achieve mean square bipartite consensus based on event-triggered protocols are given. We find that the communication topology being structurally balanced and containing a spanning tree are necessary and sufficient for ensuring a mean square bipartite consensus based on event-triggered protocols.

Organization. Section 2 gives the algebraic graph preliminaries and the problem in question. Section 3 contains the main results of the paper. Section 4 applies the results to examples of MASs with six agents. Section 5 closes this paper.

Notations.$R^{n\times m}$ represents the real matrix of $n\times m$ order. 0 denotes vector or matrix whose elements are 0. $\mathbf{1}_n$ represents column vector whose elements are 1. $\operatorname{sgn}(\cdot)$ represents the sign function. $\otimes$ represents Kronecker product. For a given matrix or vector X, X^T, and $\|X\|$ represent the transpose and European norm of X, respectively. $\|X\|_F$, $\|X\|_1$, and $\|X\|_\infty$ represent the Frobenius norm, 1-norm, and ∞-norm, respectively. $\operatorname{Re}(\lambda)$ is the real part of λ.

2. Problem Statement

The communication relations among N agents are described by the signed digraph $\mathscr{G}=(\mathscr{V},\mathscr{E},\mathscr{A})$, where $\mathscr{V}=\{1,\ldots,N\}$ and $\mathscr{E}\subseteq\mathscr{V}\times\mathscr{V}$ represent the node set and the edge set, respectively. $\mathscr{A}=(a_{ij})\in R^{N\times N}$, where $a_{ij}>0$ and $a_{ij}<0$ represent cooperation and competition between agents i and j, respectively. $a_{ij}\neq 0\Longleftrightarrow(j,i)\in\mathscr{E}$. We assume that $a_{ii}=0$ and $a_{ij}a_{ji}\geq 0,\forall i,j\in\mathscr{V}$. $\mathscr{L}=\mathscr{C}_r-\mathscr{A}$ is the Laplacian matrix of $\mathscr{G}$, where $\mathscr{C}_r=\operatorname{diag}(\sum_{j=1}^N|a_{1j}|,\ldots,\sum_{j=1}^N|a_{Nj}|)$. A signed digraph $\mathscr{G}=(\mathscr{V},\mathscr{E},\mathscr{A})$ is said structurally balanced if $\mathscr{V}$ can be divided into two subsets $\mathscr{V}_1,\mathscr{V}_2,\mathscr{V}_1\cup\mathscr{V}_2=\mathscr{V},\mathscr{V}_1\cap\mathscr{V}_2=\varnothing$, such that $a_{ij}\geq 0,\forall i,j\in\mathscr{V}_p$ $(p\in\{1,2\})$, and $a_{ij}\leq 0$, $\forall i\in\mathscr{V}_p,\ j\in\mathscr{V}_q$ $(p\neq q,p,q\in\{1,2\})$. It is said structurally unbalanced otherwise.

Lemma 1 (see [12]). *If $\mathscr{G}$ is structurally balanced, Laplacian $\mathscr{L}$ of $\mathscr{G}$ has at least one zero eigenvalue and all of the nonzero eigenvalues have positive real parts. Furthermore, $\mathscr{L}$ has only one zero eigenvalue if and only if $\mathscr{G}$ has a spanning tree.*

Consider a MAS described by

$$\dot{x}_i(t)=u_i(t),\quad i=1,\ldots,N,\tag{1}$$

where $x_i(t)\in R^n$ is the state of the ith agent and $u_i(t)\in R^n$ is the control input. A signed digraph $\mathscr{G}=(\mathscr{V},\mathscr{E},\mathscr{A})$ is used to describe interactions among the N agents.

Since communication is often disturbed by measurement noise, we assume the ith agent receives information from its neighbors with measurement noise $x_j(t)+\varphi_{ji}(t)$, $j\in\mathscr{N}_i$, $i=1,\ldots,N$. In order to reduce the frequency of controller updates, we design the following event-triggered protocol for the ith agent:

$$\begin{aligned}u_i(t)=b(t)\Bigg[&\sum_{j=1}^N|a_{ij}|\left(\operatorname{sgn}(a_{ij})x_j(t_k)-x_i(t_k)\right)\\&+\sum_{j=1}^N a_{ij}\varphi_{ji}(t)\Bigg],\quad\forall t\in[t_k,t_{k+1}),\end{aligned}\tag{2}$$

where $i=1,\ldots,N$, $k=0,1,\ldots,b(t)>0$ is a piecewise continuous function. $\{\varphi_{ji}(t)\}$ is n dimensional independent standard white noise.

Remark 2. As far as we know the existing results [28, 29] for event-triggered bipartite consensus did not consider measurement noise. Here, we take noise into consideration. If we take $b(t)\equiv 1$, then (2) is reduced to the protocols in [28, 29] without measurement noise.

Let $X(t)=(x_1^T(t),\ldots,x_N^T(t))^T$ and $J=\operatorname{diag}(\varsigma_1^T(t),\ldots,\varsigma_N^T(t))^T$ be $N\times N^2$ dimensional block diagonal matrix, where $\varsigma_i^T(t)=(a_{i1},\ldots,a_{iN})$ is the ith row element of matrix $\mathscr{A}$. Then the closed-loop system is

$$\begin{aligned}\mathrm{d}X(t)=&-b(t)(\mathscr{L}\otimes I_n)X(t_k)\,\mathrm{d}t\\&+b(t)(J\otimes I_n)\,\mathrm{d}\Lambda(t),\\&\qquad t\in[t_k,t_{k+1}),\ k=0,1,\cdots\end{aligned}\tag{3}$$

where $\Lambda(t)=(\Lambda_1^T(t),\ldots,\Lambda_N^T(t))^T$ and $\Lambda_i(t)=(\Lambda_{1i}^T(t),\ldots,\Lambda_{Ni}^T(t))^T$, $i=1,\ldots,N$. For $i,j=1,\ldots,N$, $\int_0^t\varphi_{ji}(s)ds=\Lambda_{ji}(t)$ is n dimensional standard Brownian motion. Let $e(t)=(e_1(t),\ldots,e_N(t))^T$ be the measurement error, where $e_i(t)=x_i(t_k)-x_i(t)$, $t\in[t_k,t_{k+1})$, $k=0,1,\cdots$. Then (3) is changed to

$$\begin{aligned}\mathrm{d}X(t)=&-b(t)(\mathscr{L}\otimes I_n)(X(t)+e(t))\,\mathrm{d}t\\&+b(t)(J\otimes I_n)\,\mathrm{d}\Lambda(t),\\&\qquad t\in[t_k,t_{k+1}),\ k=0,1,\cdots\end{aligned}\tag{4}$$

We present the following definition of event-triggered bipartite consensus for the stochastic system.

Definition 3. Let $\mathscr{U}=\{u_i,i=1,\ldots,N\}$ be an event-triggered protocol. If for any given $X(0)\in R^{nN}$, there exist $g=(g_1,\ldots,g_N)^T\in R^N$, $g_i\in\{\pm1\}$, $i=1,\ldots,N$ and n dimensional random vector ν^*,

$$\lim_{t\longrightarrow\infty}E\left\|X(t)-g\otimes\nu^*\right\|^2=0,\tag{5}$$

where $E\|\nu^*\|^2<\infty$, $E\nu^*$ is dependent on communication relations among agents and $X(0)$, which is deterministic.

Then, event-triggered protocol $\mathscr{U}$ is called a mean square bipartite consensus protocol.

We introduce the event-triggered condition

$$\|e(t)\| \leq c_1 e^{-\alpha t}, \tag{6}$$

where $c_1 > 0, 0 < \alpha < \min_{\lambda(\mathscr{L})\neq 0}\{\mathrm{Re}(\lambda(\mathscr{L}))\}$. When the measurement error $\|e(t)\|$ is over the threshold, the controller is triggered and updates itself.

To analyze the closed-loop system in (4), we make the following assumptions:

($\mathbf{Q_1}$) $\mathscr{G} = (\mathscr{V}, \mathscr{E}, \mathscr{A})$ is structurally balanced.

($\mathbf{Q_2}$) $\mathscr{G} = (\mathscr{V}, \mathscr{E}, \mathscr{A})$ contains a spanning tree.

($\mathbf{Q_3}$) $\int_0^\infty b(s)\mathrm{d}s = \infty$.

($\mathbf{Q_4}$) $\int_0^\infty b^2(s)\mathrm{d}s < \infty$.

The following lemma plays an important role in the following section.

Lemma 4 (see [16]). *Given linear time-varying system*

$$\frac{dY_l}{dt} = -b(t) F_l^\lambda Y_l, \quad l \in \mathbb{N},\ \lambda \in \mathbb{C},\ t \geq t_0 \geq 0, \tag{7}$$

where $Y_l = (y_{l1}, \ldots, y_{lR_l})^T \in R^{R_l}$ and F_l^λ is the $R_l \times R_l$ dimensional Jordan block, which λ is the diagonal element. Then the state transition matrix of (7) is $\Psi_{F_l^\lambda}(t, t_0) = e^{-\int_{t_0}^t b(s)ds F_l^\lambda}$. In addition, we can obtain $\lim_{t\longrightarrow\infty} \Psi_{F_l^\lambda}(t, t_0) = 0$ if $\int_0^\infty b(s)ds = \infty$ and $\mathrm{Re}(\lambda) > 0$.

Lemma 5. *If the event-triggered protocol (2) is a mean square bipartite consensus protocol, then $\exists g = (g_1, \ldots, g_N)^T \in R^N$, $g_i \in \{\pm 1\}$, $i = 1, \ldots, N$, and $\theta = (\theta_1, \ldots, \theta_N)^T \in R^N$, such that $\lim_{t\longrightarrow\infty} \Psi(t, 0) = g\theta^T \otimes I_n$, where $\Psi(t, 0)$ is the state transition matrix of (4).*

Proof. From the above condition, Definition 3 implies that for any given initial state $X(0)$, there exist a vector g and a random vector ν^* so that $\lim_{t\longrightarrow\infty} E\|X(t) - g \otimes \nu^*\|^2 = 0$. Obviously,

$$\begin{aligned} X(t) = {} & \Psi(t, 0) X(0) \\ & - \int_0^t b(s) \Psi(t, s) (\mathscr{L} \otimes I_n) e(s)\,\mathrm{d}s \\ & + \int_0^t b(s) \Psi(t, s) (J \otimes I_n)\,\mathrm{d}\Lambda(s). \end{aligned} \tag{8}$$

Without loss of generality, we assume $\int_0^t b(s)\Psi(t,s)(\mathscr{L} \otimes I_n)e(s)\mathrm{d}s$ and $\int_0^t b(s)\Psi(t,s)(J \otimes I_n)\mathrm{d}\Lambda(s)$ converge to Y^* and Z^* in mean square sense, respectively. Then,

$$\begin{aligned} g \otimes E\nu^* &= \lim_{t\longrightarrow\infty} EX(t) \\ &= (\Psi_\infty \otimes I_n) X(0) - EY^* + EZ^*, \end{aligned} \tag{9}$$

where $\lim_{t\longrightarrow\infty} \Psi(t, 0) = \Psi_\infty \otimes I_n$. According to Definition 3 and the arbitrariness of $X(0)$, one obtains $(\Psi_\infty \otimes I_n)X(0) = g \otimes Ed^*$, where $Ed^* \in R^N$.

Let $\Psi_\infty = (\varrho_1, \ldots, \varrho_N)$. Then, all elements of ϱ_i have the same absolute value. The same applies for $\sum_{j=1}^N (\varrho_j \otimes \tau_j)$, where $\tau_j \in R^n$, $j = 1, \ldots, N$. If $\Psi_\infty = 0$, then by making $\theta = 0$, Lemma 5 holds. If Ψ_∞ has at least one nonzero column, without loss of generality, we assume $\varrho_1 \neq 0$. Then $\varrho_1 = \theta_1 g$. Without loss of generality, we assume $\theta_1 > 0$. For any $\alpha, \beta \in R^n$, $a, b \in R$, $a, b \neq 0$, $a\alpha + b\beta \neq \pm(a\alpha - b\beta)$. If $\varrho_j \neq 0$ for some $j \neq 1$, then all n dimensional components of $\varrho_j \otimes \alpha - \varrho_1 \otimes \beta$ have the same modulus if and only if $\varrho_j = \theta_j g$, $\theta_j > 0$. If $\varrho_j = 0$, we have $\varrho_j = \theta_j g$ by taking $\theta_j = 0$. Then $\Psi_\infty = (\varrho_1, \ldots, \varrho_N) = (\theta_1 g, \ldots, \theta_N g) = g(\theta_1, \ldots, \theta_n)$. In addition, $\lim_{t\longrightarrow\infty} \Psi(t, 0) = \Psi_\infty \otimes I_N$, so $\lim_{t\longrightarrow\infty} \Psi(t, 0) = g(\theta_1, \theta_2, \ldots, \theta_N) \otimes I_n = g\theta^T \otimes I_n$. ☐

Lemma 6. *If* ($\mathbf{Q_1}$) − ($\mathbf{Q_4}$) *hold, then for any given initial state $X(0)$, there is a random vector X^* such that $X(t)$ converges to X^* in mean square sense, i.e., $\lim_{t\longrightarrow\infty} E\|X(t) - X^*\|^2 = 0$.*

Proof. If ($\mathbf{Q_1}$) and ($\mathbf{Q_2}$) hold, then Laplacian $\mathscr{L}$ has exactly one zero eigenvalue and all nonzero eigenvalues have positive real parts by Lemma 1. Thus, there exists an invertible matrix D, such that

$$D^{-1}\mathscr{L}D = F = \mathrm{diag}\left(0, F_2, \ldots, F_\gamma\right), \tag{10}$$

where F_i $(i = 2, \ldots, \gamma)$ is the $R_i \times R_i$ dimensional Jordan block, which λ_i is the diagonal element, and $R_2 + \cdots + R_\gamma = N - 1$. Obviously, $\lambda_2, \ldots, \lambda_\gamma$ are eigenvalues of $\mathscr{L}$ and $\mathrm{Re}(\lambda_i) > 0$, $i = 2, \ldots, \gamma$.

Since $\Psi(t, t_0)$ $(t_0 \geq 0)$ is the state transition matrix of (4), $\Psi(t, t_0) = e^{-\int_{t_0}^t b(s)\mathrm{d}s\mathscr{L}} \otimes I_n$. From Lemma 4,

$$\begin{aligned} \Psi(t, t_0) = {} & (D \otimes I_n) \\ & \cdot \mathrm{diag}\left(I_n, \Psi_{F_2^{\lambda_2}}(t, t_0) \otimes I_n, \ldots, \Psi_{F_\gamma^{\lambda_\gamma}}(t, t_0) \otimes I_n\right) \\ & \cdot \left(D^{-1} \otimes I_n\right). \end{aligned} \tag{11}$$

Combining this with ($\mathbf{Q_3}$), one has

$$\lim_{t\longrightarrow\infty} \Psi(t, t_0) = \left[D\,\mathrm{diag}(1, 0, 0, \ldots, 0)\,D^{-1}\right] \otimes I_n. \tag{12}$$

Thus, there exists $T > 0$ so that for any $t \geq t_0 > 0$,

$$\max\left(\|\Psi(t, t_0)\|_1, \|\Psi(t, t_0)\|_\infty\right) \leq T < \infty. \tag{13}$$

By Itô formula, the solution of (4) is given by

$$\begin{aligned} X(t) = {} & \Psi(t, 0) X(0) \\ & - \int_0^t b(s) \Psi(t, s) (\mathscr{L} \otimes I_n) e(s)\,\mathrm{d}s \\ & + \int_0^t b(s) \Psi(t, s) (J \otimes I_n)\,\mathrm{d}\Lambda(s). \end{aligned} \tag{14}$$

By ($\mathbf{Q_4}$), one obtains that, $\forall\varepsilon > 0$, $\exists\Gamma_0 > 0$, $\int_{\Gamma_0}^{\infty} b^2(s)\mathrm{d}s < \varepsilon$. By (12), $\exists\Gamma_1 > \Gamma_0$, such that $\max(\|\Psi(t_2,t_3) - \Psi(t_1,t_3)\|_1, \|\Psi(t_2,t_3) - \Psi(t_1,t_3)\|_\infty) < \varepsilon$, $t_2 \ge t_1 > \Gamma_1$, $\forall t_3 \in [0,\Gamma_0]$.

Let $X_2(t) = \int_0^t b(s)\Psi(t,s)(\mathscr{L}\otimes I_n)e(s)\mathrm{d}s$, then by (10) and (11), one has

$$X_2(t) = \int_0^t b(s)(D\otimes I_n) \cdot \operatorname{diag}\left(I_n, \Psi_{F_2^{\lambda_2}}(t,s)\otimes I_n, \dots, \Psi_{F_\gamma^{\lambda_\gamma}}(t,s)\otimes I_n\right) \cdot \left(D^{-1}\mathscr{L}\otimes I_n\right)e(s)\,\mathrm{d}s. \tag{15}$$

By (6), (10), and direct calculation, one has $(D^{-1}\mathscr{L}\otimes I_n)e(s) = (0, \mathscr{D}_2^T(s), \dots, \mathscr{D}_N^T(s))^T$, where $\mathscr{D}_i(s)$ $(i = 2,\dots,N)$ is the linear combination of $e_1(s),\dots,e_N(s)$. By L'Hospital and direct calculation, one obtains

$$\lim_{t\to\infty}\int_0^t b(s)e^{-\lambda_i\int_s^t b(\tau)\mathrm{d}\tau}\left(\int_s^t b(\tau)\,\mathrm{d}\tau\right)^m e^{-\alpha s}\mathrm{d}s = 0, \quad m = 0,1,\dots,R_i-1;\ i = 2,\dots,\gamma. \tag{16}$$

Noticing that $\Psi_{F_i^{\lambda_i}}(t,s) = \sum_{d=0}^{\infty}((-\int_s^t b(\tau)\mathrm{d}\tau)^d (F_i^{\lambda_i})^d/d!)$, one has $\lim_{t\to\infty}X_2(t) = 0$.

Let $X_3(t) = \int_0^t b(s)\Psi(t,s)(J\otimes I_n)\mathrm{d}\Lambda(s)$, then

$$\begin{aligned} X_3(t_2) - X_3(t_1) &= \int_0^{t_2} b(s)\left[\Psi(t_2,s) - \Psi(t_1,s)\right](J\otimes I_n)\,\mathrm{d}\Lambda(s) \\ &\quad + \int_{t_1}^{t_2} b(s)\Psi(t_1,s)(J\otimes I_n)\,\mathrm{d}\Lambda(s) \triangleq X_{31} + X_{32}. \end{aligned} \tag{17}$$

Therefore $E\|X_3(t_2) - X_3(t_1)\|^2 \le 2E\|X_{31}\|^2 + 2E\|X_{32}\|^2$. It is easy to obtain

$$E\|X_{31}\|^2 = \int_0^{t_2} b^2(s)\left\|\left[\Psi(t_2,s) - \Psi(t_1,s)\right](J\otimes I_n)\right\|_F^2\mathrm{d}s. \tag{18}$$

Noting

$$\int_0^{T_0} b^2(s)\left\|\left[\Psi(t_2,s) - \Psi(t_1,s)\right](J\otimes I_n)\right\|_F^2\mathrm{d}s \le nN\left\|(J\otimes I_n)\right\|_1^2\int_0^{\infty} b^2(s)\,\mathrm{d}s \triangleq M_4\varepsilon^2 \tag{19}$$

and

$$\int_{\Gamma_0}^{t_2} b^2(s)\left\|\left[\Psi(t_2,s) - \Psi(t_1,s)\right](J\otimes I_n)\right\|_F^2\mathrm{d}s \le 4nNL^2\left\|(J\otimes I_n)\right\|_1^2\varepsilon \triangleq M_5\varepsilon, \tag{20}$$

one has $E\|X_{31}\|^2 \le M_4\varepsilon^2 + M_5\varepsilon$. Similarly, one obtains

$$\begin{aligned} E\|X_{32}\|^2 &= \int_{t_1}^{t_2} b^2(s)\left\|\Psi(t_1,s)(J\otimes I_n)\right\|_F^2\mathrm{d}s \\ &\le nNL^2\left\|(J\otimes I_n)\right\|_1^2\varepsilon \triangleq \frac{M_5}{4}\varepsilon. \end{aligned} \tag{21}$$

So $E\|X_3(t_2) - X_3(t_1)\|^2 \le 2M_4\varepsilon^2 + (5/2)M_5\varepsilon$. By Cauchy criterion and the arbitrariness of ε, there exists X_3^* such that $X_3(t)$ converges to X_3^* in mean square sense. So there exists X^* such that $X(t)$ converges to X^* in mean square sense. By (12), $X^* = [D\operatorname{diag}(1,0,\dots,0)D^{-1}\otimes I_n]X(0) + X_3^*$. $\square$

3. Main Results

In this section, we give necessary and sufficient conditions for the proposed event-triggered protocols to guarantee a mean square bipartite consensus.

Theorem 7. *The event-triggered protocol in (2) is a mean square bipartite consensus protocol for the system in (1) if and only if ($\mathbf{Q_1}$)-($\mathbf{Q_4}$) hold.*

Proof (sufficiency).

(S.1) Construct a Bipartition for the MAS. By ($\mathbf{Q_3}$), $\mathscr{V}$ can be decomposed into two disjoint subsets $\mathscr{V}_{v_1}, \mathscr{V}_{v_2}$, $\mathscr{V}_{v_1}\cup\mathscr{V}_{v_2} = \mathscr{V}$, $\mathscr{V}_{v_1}\cap\mathscr{V}_{v_2} = \varnothing$, and $a_{st} \le 0$ for $s \in \mathscr{V}_f, t \in \mathscr{V}_g$, $f \ne g$, $f,g \in \{v_1,v_2\}$, and $a_{st} \ge 0$ for $s,t \in \mathscr{V}_p$, $p \in \{v_1,v_2\}$. Without loss of generality, we assume $\mathscr{V}_{v_1} = \{1,\dots,m\}$, $\mathscr{V}_{v_2} = \{m+1,\dots,N\}$. Let $g_i = 1$ for $i \in \mathscr{V}_{v_1}$ and $g_j = -1$ for $j \in \mathscr{V}_{v_2}$. By definition, one has $\mathscr{L}g = 0$, where $g = (g_1,\dots,g_N)^T$.

(S.2) Prove $\lim_{t\to\infty}E\|X(t) - g\otimes v^*\|^2 = 0$. From Lemma 6, $\exists X^*$, $\lim_{t\to\infty}E\|X(t) - X^*\|^2 = 0$. Without loss of generality, we assume $\lim_{t\to\infty}E\|x_1(t) - v^*\|^2 = 0$. Next, we will prove $\lim_{t\to\infty}E\|x_i(t) - g_i v^*\|^2 = 0$, $i = 2,\dots,N$.

Let $\phi(t) = (\phi_2^T(t),\dots,\phi_N^T(t))^T$, where $\phi_i(t) = x_i(t) - g_i x_1(t)$, $i = 2,\dots,N$. Now we prove that $\lim_{t\to\infty}E\|\phi(t)\|^2 = 0$. For this purpose, We assume $\tilde{\phi}(t) \triangleq (Q\otimes I_n)X(t)$, where

$$Q \triangleq \left(\begin{array}{c|ccc|ccc} \dfrac{1}{N} & \dfrac{1}{N} & \cdots & \dfrac{1}{N} & -\dfrac{1}{N} & \cdots & -\dfrac{1}{N} \\ -\mathbf{1}_{m-1} & & I_{m-1} & & & 0 & \\ -\mathbf{1}_{N-m} & & 0 & & & -I_{N-m} & \end{array}\right) = \begin{pmatrix} \dfrac{1}{N}g^T \\ Q_2 \end{pmatrix}. \tag{22}$$

Then $\tilde{\phi}(t) = (\chi^T(t), \phi^T(t))^T$, where $\chi(t) \triangleq (1/N)(g^T\otimes I_n)X(t)$. Since

$$Q\mathscr{L}Q^{-1} = \begin{pmatrix} 0 & \varpi^T \\ 0 & L_2 \end{pmatrix}, \tag{23}$$

by (4), one has

$$\begin{aligned} d\phi(t) &= -b(t)\left(L_2 \otimes I_n\right)\phi(t)\,\mathrm{d}t \\ &\quad - b(t)\left(Q_2 L \otimes I_n\right)e(t)\,\mathrm{d}t \\ &\quad + b(t)\left(Q_2 J \otimes I_n\right)\mathrm{d}\Lambda(t). \end{aligned} \tag{24}$$

By (23), $S^{-1}L_2S = \operatorname{diag}(F_2,\ldots,F_\gamma)$, where S is invertible and $F_2,\ldots,F_\gamma$ are given in (10). The state transition matrix of the system in (24) is

$$\begin{aligned} \Psi_2(t,t_0) &= \left(S \otimes I_n\right) \\ &\quad \cdot \left[\operatorname{diag}\left(\Psi_{F_2^{\lambda_2}}(t,t_0)\otimes I_n,\ldots,\Psi_{F_\gamma^{\lambda_\gamma}}(t,t_0)\otimes I_n\right)\right] \\ &\quad \cdot \left(S^{-1}\otimes I_n\right), \end{aligned} \tag{25}$$

where $\Psi_{F_q^{\lambda_q}}(t,t_0)$, $q = 2,\ldots,\gamma$ are defined as in Lemma 4. Hence, $\lim_{t\longrightarrow\infty}\Psi_2(t,t_0) = 0$, i.e., $\forall\varepsilon > 0$, $\exists\Gamma_2 > \Gamma_1$, such that $\|\Psi_2(t,t_0)\| < \varepsilon$, $\forall t > \Gamma_2$. Furthermore, $\exists T_{L_2} > 0$, such that, $\forall t > t_0 \geq 0$, $\max(\|\Psi_2(t,t_0)\|_1, \|\Psi_2(t,t_0)\|_\infty) \leq T_{L_2} < \infty$.

By Itô formula, it can be seen that the state of the system in (24) can be described as

$$\begin{aligned} \phi(t) &= \Psi_2(t,0)\phi(0) \\ &\quad - \int_0^t b(s)\Psi_2(t,s)\left(Q_2L\otimes I_n\right)e(s)\,\mathrm{d}s \\ &\quad + \int_0^t b(s)\Psi_2(t,s)\left(Q_2J\otimes I_n\right)\mathrm{d}\Lambda(s). \end{aligned} \tag{26}$$

Therefore,

$$\begin{aligned} E\|\phi(t)\|^2 &= \|\Psi_2(t,0)\phi(0)\|^2 - 2\phi^T(0)\Psi_2^T(t,0) \\ &\quad \cdot \int_0^t b(s)\Psi_2(t,s)\left(Q_2L\otimes I_n\right)e(s)\,\mathrm{d}s \\ &\quad + \left\|\int_0^t b(s)\Psi_2(t,s)\left(Q_2L\otimes I_n\right)e(s)\,\mathrm{d}s\right\|^2 \\ &\quad + \int_0^t b^2(s)\left\|\Psi_2(t,s)\left(Q_2J\otimes I_n\right)\right\|_F^2\mathrm{d}s, \end{aligned} \tag{27}$$

and hence,

$$\begin{aligned} &E\|\phi(t)\|^2 \\ &\quad \leq 2\|\Psi_2(t,0)\phi(0)\|^2 \\ &\quad + 2\left\|\int_0^t b(s)\Psi_2(t,s)\left(Q_2L\otimes I_n\right)e(s)\,\mathrm{d}s\right\|^2 \\ &\quad + \int_0^t b^2(s)\left\|\Psi_2(t,s)\left(Q_2J\otimes I_n\right)\right\|_F^2\mathrm{d}s. \end{aligned} \tag{28}$$

Since

$$\begin{aligned} &\left\|\int_0^t b(s)\Psi_2(t,s)\left(Q_2L\otimes I_n\right)e(s)\,\mathrm{d}s\right\|^2 \\ &\leq \int_0^t b^2(s)\|\Psi_2(t,s)\|^2\mathrm{d}s\int_0^t \left\|\left(Q_2L\otimes I_n\right)\right\|^2\|e(s)\|^2\mathrm{d}s \end{aligned} \tag{29}$$

and

$$\begin{aligned} &\int_0^t \left\|\left(Q_2L\otimes I_n\right)\right\|^2\|e(s)\|^2\mathrm{d}s \\ &\quad \leq \left\|\left(Q_2L\otimes I_n\right)\right\|^2\int_0^t c_1^2e^{-2\alpha s}\mathrm{d}s \leq \frac{c_1^2}{2\alpha}\left\|\left(Q_2L\otimes I_n\right)\right\|^2, \end{aligned} \tag{30}$$

there exists $\beta_3 > 0$ such that $\int_0^t \|(Q_2L\otimes I_n)\|^2\|e(s)\|^2\mathrm{d}s \leq \beta_3$. Then

$$\begin{aligned} &\left\|\int_0^t b(s)\Psi_2(t,s)\left(Q_2L\otimes I_n\right)e(s)\,\mathrm{d}s\right\|^2 \\ &\quad \leqslant \beta_3\int_0^{\Gamma_0} b^2(s)\|\Psi_2(t,s)\|^2\mathrm{d}s \\ &\quad + \beta_3\int_{\Gamma_0}^t b^2(s)\|\Psi_2(t,s)\|^2\mathrm{d}s. \end{aligned} \tag{31}$$

Since $\int_0^{\Gamma_0} b^2(s)\|\Psi_2(t,s)\|^2\mathrm{d}s = \|\Psi_2(t,\eta)\|^2\int_0^{\Gamma_0} b^2(s)\mathrm{d}s \leq \|\Psi_2(t,\eta)\|^2\int_0^\infty b^2(s)\mathrm{d}s$, where $\eta \in (0,\Gamma_0)$ and $\int_{\Gamma_0}^\infty b^2(s)\mathrm{d}s < \varepsilon$, one has $\int_{\Gamma_0}^t b^2(s)\|\Psi_2(t,s)\|^2\mathrm{d}s \leq T_{L_2}^2\varepsilon \triangleq M_6\varepsilon$. Therefore, $\forall t > T_2$

$$\begin{aligned} &\left\|\int_0^t b(s)\Psi_2(t,s)\left(Q_2L\otimes I_n\right)e(s)\,\mathrm{d}s\right\|^2 \\ &\quad \leq \beta_3\|\Psi_2(t,\eta)\|^2\int_0^\infty b^2(s)\,\mathrm{d}s + M_6\beta_3\varepsilon \\ &\quad \leq \varepsilon^2\beta_3\int_0^\infty b^2(s)\,\mathrm{d}s + M_6\beta_3\varepsilon. \end{aligned} \tag{32}$$

From $\int_{\Gamma_0}^\infty b^2(s)ds < \varepsilon$, one gets

$$\begin{aligned} &\int_{\Gamma_0}^t b^2(s)\left\|\Psi_2(t,s)\left(Q_2J\otimes I_n\right)\right\|_F^2\mathrm{d}s \\ &\quad < nNT_{L_2}^2\left\|\left(Q_2J\otimes I_n\right)\right\|\,|_1^2\,\varepsilon \triangleq M_7\varepsilon. \end{aligned} \tag{33}$$

Combining this with

$$\begin{aligned} &\int_0^{\Gamma_0} b^2(s)\left\|\Psi_2(t,s)\left(Q_2J\otimes I_n\right)\right\|_F^2\mathrm{d}s \\ &\quad \leq nN\|\Psi_2(t,\xi)\|_1^2\left\|\left(Q_2J\otimes I_n\right)\right\|_1^2\int_0^\infty b^2(s)\,\mathrm{d}s, \\ &\qquad \xi \in \left(0,\Gamma_0\right), \end{aligned} \tag{34}$$

one has

$$\begin{aligned} E\|\phi(t)\|^2 &\leq 2\|\phi(0)\|^2\varepsilon^2 + 2\beta_3\varepsilon^2\int_0^\infty b^2(s)\,\mathrm{d}s \\ &\quad + nN\varepsilon^2\left\|\left(Q_2J\otimes I_n\right)\right\|_1^2\int_0^\infty b^2(s)\,\mathrm{d}s \\ &\quad + 2M_6\beta_3\varepsilon + M_7\varepsilon. \end{aligned} \tag{35}$$

By the arbitrariness of ε, one gets $\lim_{t\longrightarrow\infty}E\|\phi(t)\|^2 = 0$. Hence, $\lim_{t\longrightarrow\infty}E\|X(t) - g\otimes\nu^*\|^2 = 0$.

(S.3) Analyze the Statistical Characteristics of ν^*. By Lemma 6, $g\otimes\nu^* = X^* = [D\,\mathrm{diag}(1,0,0,\ldots,0)D^{-1}\otimes I_n]X(0) + X_3^*$. So $g\otimes E\nu^* = EX^* = [D\,\mathrm{diag}(1,0,0,\ldots,0)D^{-1}\otimes I_n]X(0)$.

We assume m_r, m_l^T represent the first column of D and the first row of D^{-1}, respectively. Then, $g\otimes E\nu^* = (m_r m_l^T\otimes I_n)X(0)$. Since $D^{-1}\mathscr{L}D = F$, $\mathscr{L}D = DF$, and $D^{-1}\mathscr{L} = FD^{-1}$, $\mathscr{L}m_r = 0$ and $m_l^T\mathscr{L} = 0$. By (**S.1**), $\mathscr{L}g = 0$. Therefore, $m_r = \kappa g$ ($\kappa\in R$ and $\kappa\neq 0$) and $g\otimes E\nu^* = g\otimes[\kappa(m_l^T\otimes I_n)X(0)]$. Then $E\nu^* = \kappa(m_l^T\otimes I_n)X(0)$. Clearly, m_l is concerned with communication topology. Thus, $E\nu^*$ is determined by $X(0)$ and communication topology of MASs.

It is easy to obtain that $\Psi(,)$ is uniformly bounded. Therefore, $\forall\varepsilon > 0$, $\exists\Gamma_3 > \Gamma_2$,

$$\begin{gathered}\int_{\Gamma_3}^{\infty} b^2(s)\left\|\Psi(t,s)(J\otimes I_n)(J\otimes I_n)^T\Psi^T(t,s)\right\|\mathrm{d}s < \varepsilon,\\ \int_{\Gamma_3}^{\infty} b^2(s)\left\|\left(m_r m_l^T\otimes I_n\right)(J\otimes I_n)(J\otimes I_n)^T\right.\\ \left.\cdot\left(m_r m_l^T\otimes I_n\right)^T\right\|\mathrm{d}s < \varepsilon.\end{gathered}\tag{36}$$

Let $X_4(t) = \int_0^{\Gamma_3} b^2(s)\Psi(t,s)(J\otimes I_n)(J\otimes I_n)^T\Psi^T(t,s)\mathrm{d}s$. Then for any $t > \Gamma_4$, $\|X_4(t) - \int_0^{\Gamma_3} b^2(s)(m_r m_l^T\otimes I_n)(J\otimes I_n)(J\otimes I_n)^T(m_r m_l^T\otimes I_n)^T\mathrm{d}s\| < \varepsilon$. This together with (36) leads to $\lim_{t\longrightarrow\infty}\int_0^t b^2(s)\Psi(t,s)(J\otimes I_n)(J\otimes I_n)^T\Psi^T(t,s)\mathrm{d}s = \int_0^\infty b^2(s)(m_r m_l^T JJ^T m_l m_r^T\otimes I_n)\mathrm{d}s = \Theta(gg^T\otimes I_n)$, where $\Theta = \kappa^2(m_l^T JJ^T m_l)\int_0^\infty b^2(s)\mathrm{d}s$. Combining this with $D(X^*) = \lim_{t\longrightarrow\infty}\int_0^t b^2(s)\times\Psi(t,s)(J\otimes I_n)(J\otimes I_n)^T\Psi^T(t,s)\mathrm{d}s$, one gets $D(\nu^*) = \Theta I_n$. Therefore, $E\|\nu^*\|^2 < \infty$. By Definition 3, the sufficiency is established.

Necessity.

(B.1) Prove (Q_3), Namely, $\int_0^\infty b(s)\boldsymbol{d}s = \infty$. By contradiction, we assume that $(\mathbf{Q_3})$ does not hold. Then, $\exists\bar{\epsilon} > 0$, $\int_0^\infty b(s)\mathrm{d}s = \bar{\epsilon}$, and $\lim_{t\longrightarrow\infty}e^{-\int_0^t b(s)\mathrm{d}s\mathscr{L}} = e^{-\bar{\epsilon}\mathscr{L}}$. Therefore, $\mathrm{rank}(\lim_{t\longrightarrow\infty}\Psi(t,0)) = \mathrm{rank}(e^{-\bar{\epsilon}\mathscr{L}}\otimes I_n) = nN$. However, by Lemma 5, $\lim_{t\longrightarrow\infty}\Psi(t,0) = g\theta^T\otimes I_n$ and $\mathrm{rank}(\lim_{t\longrightarrow\infty}\Psi(t,0))\leq n$. This is a contradiction. So $(\mathbf{Q_3})$ holds.

(B.2) Prove That Laplacian Matrix $\mathscr{L}$ Has Exactly One Zero Eigenvalue. By contradiction, we assume that 0 is not an eigenvalue of $\mathscr{L}$. Then all the eigenvalues of $\mathscr{L}$ have positive real part and $-\mathscr{L}$ is a Hurwitz matrix. By $(\mathbf{Q_3})$ and Lemma 4, $\lim_{t\longrightarrow\infty}\Psi(t,0) = 0$. Combining this with Lemma 5, one has $g\otimes E\nu^* = EZ^* - EY^*$. Since EZ^* and EY^* are independent of $X(0)$, $E\nu^*$ is independent of $X(0)$. This contradicts Definition 3. So 0 is an eigenvalue of $\mathscr{L}$.

Let F_1^0 be a Jordan block with eigenvalue 0. Then it is 1 dimensional. Otherwise, we assume F_1^0 is R_1 dimensional and $R_1 > 1$. Then, by $(\mathbf{Q_3})$ and the definition of matrix exponent function, one gets that $\lim_{t\longrightarrow\infty}e^{-\int_0^t b(s)\mathrm{d}sF_1^0}$ does not exist, and hence, $\lim_{t\longrightarrow\infty}\Psi(t,0)$ does not exist. This contradicts Lemma 5. So F_1^0 is 1 dimensional.

Let algebra multiplicity of eigenvalue 0 be w. Then $w = 1$. Otherwise, $w > 1$. Take $w = 2$ as an example. Since each Jordan block corresponding to eigenvalue 0 is 1 dimensional,

$$\begin{aligned}\Psi_\infty\otimes I_n &\triangleq \lim_{t\longrightarrow\infty}\Psi(t,0) = \lim_{t\longrightarrow\infty}e^{-\int_0^t b(s)\mathrm{d}s\mathscr{L}}\otimes I_n\\ &= D\,\mathrm{diag}(1,1,0,\ldots,0)\,D^{-1}\otimes I_n.\end{aligned}\tag{37}$$

Thus, $\mathrm{rank}(\Psi_\infty) = 2$. This contradicts $\mathrm{rank}(\Psi_\infty)\leq 1$ from Lemma 6. So Laplacian $\mathscr{L}$ has exactly one zero eigenvalue.

(B.3) Prove (Q_1) and (Q_2). By (**B.2**) and $(\mathbf{Q_3})$, one has (12). By Lemma 5, one gets

$$D\,\mathrm{diag}(1,0,0,\ldots,0)\,D^{-1} = g\theta^T.\tag{38}$$

Noticing that m_r is the first column of D, one has $\mathscr{L}m_r = 0$. By (38), one obtains $m_r = g\kappa^*$, where $\kappa^* = \theta^T m_r\in R$. Then, $\mathscr{L}g = 0$. By the definition of $\mathscr{L}$, for any j, we obtain $g_j\sum_{k\neq j}|a_{jk}| = \sum_{k\neq j}g_k a_{jk}$, $j = 1,\ldots,N$. Since $g_j = \pm 1$ and $g_j^2 = 1$, $j = 1,2,\ldots,N$, $\sum_{k\neq j}|a_{jk}| = \sum_{k\neq j}g_j g_k a_{jk}$. So $g_j g_k a_{jk} = |a_{jk}|\geq 0$. Let $V_1 = \{j\mid g_j = 1\}$ and $V_2 = \{j\mid g_j = -1\}$, then $V_1\cap V_2 = \varnothing$, $V_1\cup V_2 = V$. If $j\in\mathscr{V}_p$, $p\in\{1,2\}$, then $a_{jk}\geq 0$, $k\in\mathscr{V}_p$ or $a_{jk}\leq 0$, $k\in\mathscr{V}_r$, $r\neq p$, $r\in\{1,2\}$. By definition, $\mathscr{G}$ is structurally balanced, that is, $(\mathbf{Q_1})$ holds.

By (**B.2**) and $(\mathbf{Q_1})$, Lemma 1 implies that $(\mathbf{Q_2})$ holds.

(B.4) Prove (Q_4). Assume $\int_0^\infty b^2(s)\mathrm{d}s = \infty$. Due to the first row of D^{-1} which is m_l^T, $m_l^T\mathscr{L} = 0$. By (4), we obtain $\mathrm{d}((m_l^T\otimes I_n)X(t)) = b(t)(m_l^T\otimes I_n)(J\otimes I_n)\mathrm{d}\Lambda(t)$, *i.e.*,

$$\begin{aligned}&\left(m_l^T\otimes I_n\right)X(t)\\ &\quad = \left(m_l^T\otimes I_n\right)X(0)\\ &\qquad + \left(m_l^T\otimes I_n\right)(J\otimes I_n)\int_0^t b(s)\,\mathrm{d}\Lambda(s).\end{aligned}\tag{39}$$

From Definition 3, it is known that $X(t)$ converges to $g\otimes\nu^*$ in mean square sense, where $E\|\nu^*\|^2 < \infty$. Thus, when $t\longrightarrow\infty$, $(m_l^T\otimes I_n)(J\otimes I_n)\int_0^t b(s)\mathrm{d}\Lambda(s)$ converges to a random variable X_m in mean square sense with $E\|X_m\|^2 < \infty$. Then $\lim_{t\longrightarrow\infty}E\|(m_l^T\otimes I_n)(J\otimes I_n)\int_0^t b(s)\mathrm{d}\Lambda(s)\|^2 = \lim_{t\longrightarrow\infty}\mathrm{tr}(m_l^T JJ^T m_l\otimes I_n)\int_0^t b^2(s)\mathrm{d}s = \infty$. This leads to a contradiction. So $(\mathbf{Q_4})$ holds. □

Remark 8. From Theorem 7 it can be seen that under $(\mathbf{Q_1})$-$(\mathbf{Q_4})$ the event-triggered protocol in (2) ensures agents converging to ν^* or $-\nu^*$ under measurement noise.

Remark 9. From Theorem 7 one sees that to guarantee the mean square bipartite consensus, $(\mathbf{Q_1})$-$(\mathbf{Q_2})$ are requirements for time-varying gain $b(t)$ while $(\mathbf{Q_3})$-$(\mathbf{Q_4})$ are the weakest connectivity assumptions.

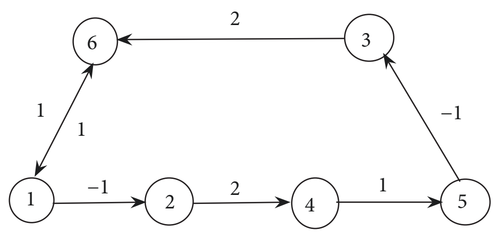

Figure 1: Communication graph $\mathcal{G}$ among the 6 agents.

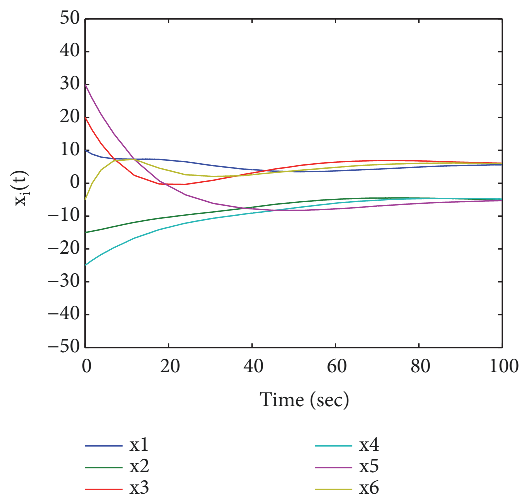

Figure 2: State trajectories of six agents.

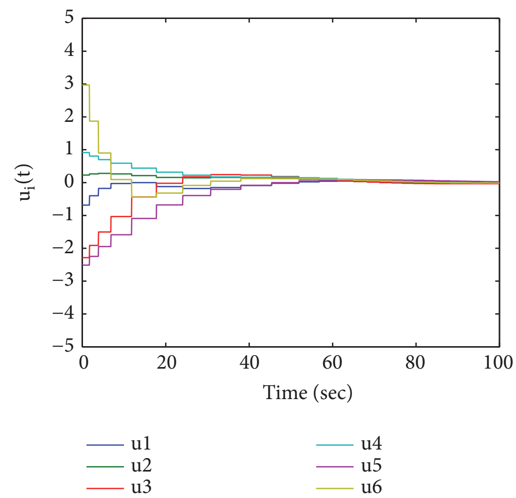

Figure 3: Control inputs of six agents.

4. Numerical Simulation

To demonstrate the developed result in the preceding, we consider an MAS of six agents, whose dynamics satisfy the system in (1). The communication graph that connects the six agents is illustrated in Figure 1. Clearly, $\mathcal{V} = \{1, \ldots, 6\}$, $\mathcal{A} = (a_{ij})$, $a_{16} = a_{61} = a_{54} = 1$, $a_{21} = a_{35} = -1$, and $a_{42} = a_{63} = 2$ in $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$. From Figure 1, $\mathcal{G}$ satisfies $(\mathbf{Q_1})$ and $(\mathbf{Q_2})$. Furthermore, all eigenvalues of Laplacian $\mathcal{L}$ are $\lambda_1 = 0$, $\lambda_2 = 0.6733 + 0.9192\jmath$, $\lambda_3 = 0.6733 - 0.9192\jmath$, $\lambda_4 = 2.0887 + 0.7157\jmath$, $\lambda_5 = 2.0887 - 0.7157\jmath$, and $\lambda_6 = 3.4760$ ($\jmath^2 = -1$). Obviously, $\min_{\lambda(\mathcal{L}) \neq 0}\{\mathrm{Re}\,\lambda(\mathcal{L})\} = 0.6733$. The initial state of the MAS is given by $X(0) = (10, -15, 20, -25, 30, -5)$. Choose $b(t) = \ln(t+1)/(t+1)$. By direct calculation we know that $b(t)$ satisfies $(\mathbf{Q_3})$-$(\mathbf{Q_4})$. Assume event-triggered condition (6) is satisfied by taking $c_1 = 1.2$ and $\alpha = 0.6$. Applying protocol (2) to the system in (1), we get the six agents' state trajectories. As shown in Figure 2 one can see that the states of agents 1, 3, and 6 converge to 5 in mean square sense while the states of agents 2, 4, and 5 converge to -5 in mean square sense. Thus, mean square bipartite consensus is achieved with event-triggered protocol (2). On the other hand, from Figure 3 we know that the inputs are constants between the event triggering time interval. Moreover, from Figure 4, it can be seen that the absolute value of the measurement error of each agent converges to zero. This means that the MAS does not exhibit Zeno behavior.

5. Conclusion

Mean square bipartite consensus problem of single-integrator MASs is investigated in the context of event-triggered control and measurement noise. By using time-varying gain, an event-triggered bipartite consensus protocol is proposed under measurement noise, with which the controller update frequency is reduced. With given necessary and sufficient conditions on protocol gain and communication topology, the MAS is proved to achieve event-triggered bipartite consensus. The simulation shows that the system will not show Zeno behavior.

Data Availability

The Matlab based models used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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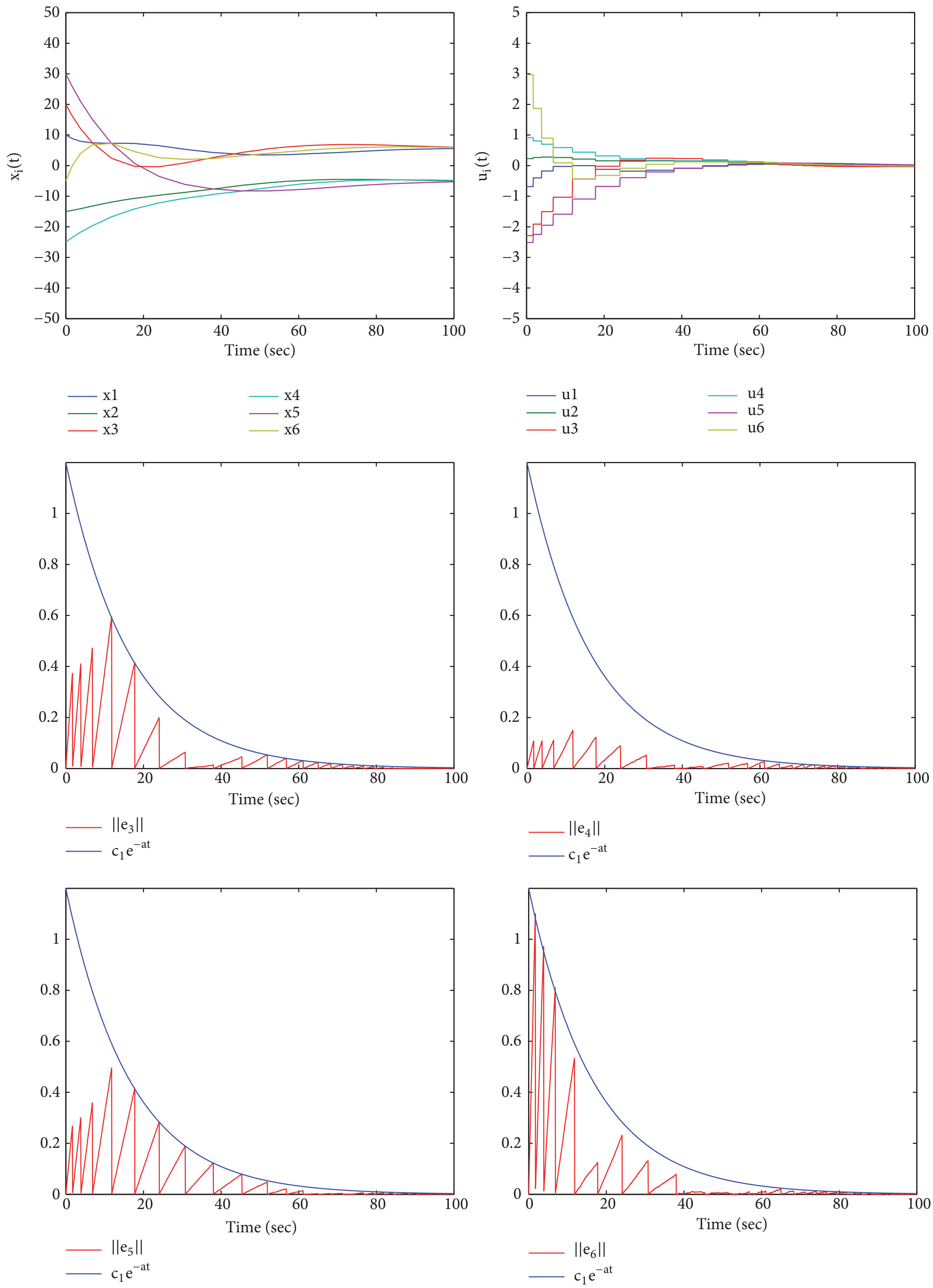

Figure 4: The evolution of error norm.

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