

Channel-Aware Scheduling for Multiple Control Systems with Packet-Based Control over Collision Channels

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Abstract—We consider a wireless control architecture with multiple control systems communicating over two shared collision channels. Each sensor accesses the channel randomly and a scheduler at the controller side decides which controller is permitted to access the channel. We design a packet-based model predictive controller and obtain the packet transmission success probability demands of stability. The channel-aware transmission strategy of each sensor is studied in the non-cooperative game theory framework. We also characterize the Nash equilibrium and design a decentralized channel access mechanism to achieve the Nash equilibrium. The effectiveness of our results is demonstrated by a numerical simulation.

I. INTRODUCTION

Wireless control systems (WCS) are a kind of feedback control systems where the information transmitted between the sensors, controllers and actuators via wireless channels. The architecture of WCS has advantages in terms of low cost, fast deployment and increased flexibility, etc. However, the introduction of the wireless medium may lead to some new challenges because wireless medium is inherently unreliable. For example, the wireless channels are frequently subject to time-varying fading and interference, which may result in packet losses [1]. Since the wireless medium is shared, multiple control loops access the channel simultaneously will cause packet collision. In these scenarios, the scheduling is indispensable in accessing the channel and the compensation for information loss is important in control, especially for the scarce communication resource, see e.g. [2], [3], [4].

According to the architecture of the WCS, the scheduling mechanism can be either centralized or decentralized [4]. For centralized scheduling mechanism, there usually exists a central scheduler to decide which device can access the wireless channel or whether to transmit the information or not at each time slot. The decision may be deterministic [5], random [6], state-based [2], [7] or channel-aware [8]. In general, the social optimal can be achieved and the resource utilization can be maximized by the centralized scheduling mechanism. In contrast to the centralized scheduling

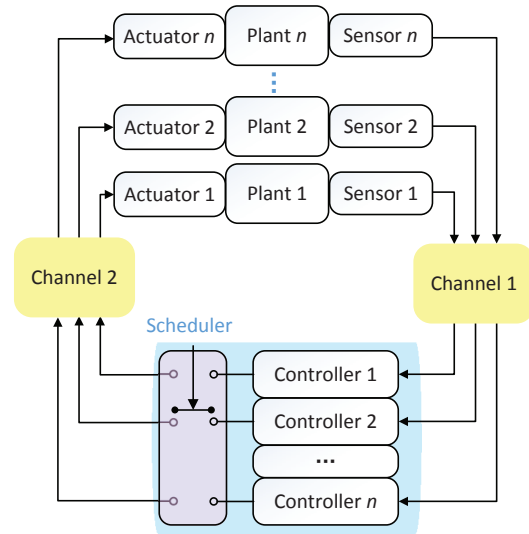


Fig. 1. Architecture of wireless control systems with multiple systems

mechanism that requires the coordination among devices or communication between scheduler and devices, we are interested in the decentralized scheduling mechanism which allows each device to make decision independently. This mechanism commonly results in efficiency loss [9]. For example, for the channel access problem studied in this paper, the decentralized mechanism will inevitably cause the packet collision and deteriorate the control performance. Our goal is to design the scheduling strategy and packet loss compensation mechanism to guarantee the stability of each system. The research for this topic has not been fully investigated and the existing literatures mainly focus on the access mechanism, such as ALOHA-like scheme [10] and random access [11], [12]. In all the related works, the packet loss arisen from the scheduling strategy has not been compensated, which will in turn bring the conservatism in designing a scheduling strategy.

Inspired by the above works, a channel-aware decentralized access strategy for each sensor, a centralized selection strategy for scheduler, and a packet-based model predictive controller (MPC) that used to actively compensate the packet losses incurred by the scheduling and other interference, are designed to minimize the average transmission power while guaranteeing the stability. The architecture of the WCS with multiple systems is illustrated in Fig.1. Based on the designed packet-based MPC, we analyze the stability of the closed-loop system and explore the two channels' packet transmission success probabilities requirements of stability.

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We design the access strategy for each sensor in the non-cooperative game theory framework, and also design the centralized controller selection strategy for the scheduler. A decentralized channel access mechanism based on the better-response dynamic is applied to achieve the Nash equilibrium.

The rest of the paper is organized as follows. The architecture of the WCS is introduced in Section II. The control strategy with the stability analysis, the access strategy for sensor and selection strategy for scheduler are studied in Section III and Section IV. In Section V, a decentralized channel access mechanism is designed. A numerical example is shown in Section VI to illustrate the effectiveness of our results. Finally, Section VII concludes the paper.

Notations. Throughout this paper, \mathbb{R}^n represents the n -dimensional Euclidean space. 0_n and I_n stand for the $n \times n$ -dimensional zero matrix and identity matrix, respectively. For a vector x , x^T represents the transpose of x and $\|x\|$ means the Euclidean norm of x . Sequence $\{z(k)\}_0^\infty$ represents $\{z(0), z(1), \dots\}$. We use E to denote mathematical expectation.

II. PROBLEM DESCRIPTION

Consider a control architecture, see in Fig.1, where n independent plants are controlled over two shared wireless collision channels named sensor-controller channel (channel 1) and controller-actuator channel (channel 2), respectively. For the collision channel, a state/control packet transmission can be successful only if no other sensors/controllers attempt to access the channel simultaneously. Therefore, the scheduling for the sensors and controllers are necessary when access the channels. Suppose that all sensors are spatially distributed, and the communication among all sensors are expensive. We thus consider the decentralized architecture, i.e., each sensor measures the states and independently decides whether to transmit or not. In contrast, all controllers are assumed to be located at one place, which makes it possible to implement a central scheduler to choose a controller to access the channel when more than one controller have transmission requirements. By this mean, the packet collision over channel 2 is avoided. Note that the controller may still compute the new control input even the new state information is not received. Once controller computes the control signal, it has the transmission requirement.

Each plant i is modelled as the following nonlinear system:

$$x_i(k+1) = f_i(x_i(k), u_i(k), w_i(k)), i = 1, 2, \dots, n \quad (1)$$

where $f(0, 0, 0) = 0$, $x_i(k) \in \mathbb{R}^{n_i}$ and $u_i(k) \in \mathbb{U} \subset \mathbb{R}^{r_i}$ are the state and control input of plant i , respectively. $w_i(k)$ is the uncertain disturbance that belongs to a compact set $\mathbb{W} \subset \mathbb{R}^{m_i}$. Suppose that $0 \in \mathbb{W}$, $d = |\mathbb{W}| \triangleq \max_{w \in \mathbb{W}}(\|w\|) < \infty$.

Suppose that each sensor is battery-powered, it transmits information to controller with fixed power level. *Our goal is to design the access strategy for each sensor, the central scheduler and the controller to minimize the energy consumption of each sensor while keeping each system stable.*

In the following, we give a detailed description of the shared wireless channel model.

Channel State Information The channel state information (CSI) reflects the current channel condition and is crucial for achieving reliable communication. To take the channel 1 for example, we describe the characteristics of the shared channel in detail. At the beginning of each time slot k , every system i obtains a channel state information (CSI) $z_i(k) \in \mathcal{H}_i$ of channel 1, which is the channel fading coefficient of the channel. $\mathcal{H}_i \in \mathbb{R}$ is a finite set of possible values of CSI and its elements are denoted by $\{h_{i1}, h_{i2}, \dots, h_{im_i}\}$ with $h_{i1} < h_{i2} < \dots < h_{im_i}$. Note that although the CSI takes continuous value in practice, in this paper, the possible value are restricted to \mathcal{H}_i for easily analyzing. The set \mathcal{H}_i can be obtained by discretization method or some reasonable classifications [13]. As pointed in [14], $z_i(k)$ changes unpredictably over time due to the propagation effect. The block fading model, where $z_i(k)$ keeps constant during each transmission slot k but is independent and identically distributed across different time slots, is adopted to model the CSI [14].

Assumption 1: (i) The probability of observing a CSI signal $z \in \mathcal{H}_i$ during any time slot is denoted by $O_i(z) > 0$. (ii) The sequences $\{z_i(k)\}_{k=0}^\infty$ and $\{z_j(k)\}_{k=0}^\infty$ are independent for $i \neq j$.

Successful Decoding Probability We assume that each plant i transmit information to the remote controller with a fixed power level ξ_i , and the information is transmitted over i.i.d. block fading additive white Gaussian noise channels (AWGN). As shown in [15], if the noise power is W_i , the channel state is z_i and the transmission power is ξ_i , then the successful decoding probability of the transmitted packet depends on the signal to noise ratio (SNR), which is defined as $SNR_i = \frac{z_i \xi_i}{W_i}$. The specific relation depends on the particular modulation and error correction code (e.g. forward error correcting (FEC) code). The successful decoding probability is denoted by $P(z_i) = \mathbb{P}(z_i, \xi_i)$, where $\mathbb{P}(\cdot)$ is an increasing function of $z_i \xi_i$. An illustration of this relation can be seen in Fig.2 in [8]. It is difficult to give the analytical expression of $\mathbb{P}(h_i, \xi_i)$, but the value can be measured in actual or simulation experiments [16].

III. CONTROL STRATEGY AND STABILITY

A. Packet-Based MPC

As stated above, there exists information loss in the above control architecture. For example, the current system state and control input may not received by the corresponding controller and actuator. The information loss is partly caused by the access strategy, packet collision and channel noise. In order to actively compensate the loss, a packet-based model predictive control method is adopted in this paper. As illustrated in Fig.2, the implementation of the packet-based MPC contains two essential components: the smart controller and the smart actuator. Due to the space limitation, we only give a brief description.

Smart Controller The smart controller of plant i consists three parts: a MPC, a state estimator and a buffer stores the control sequence.

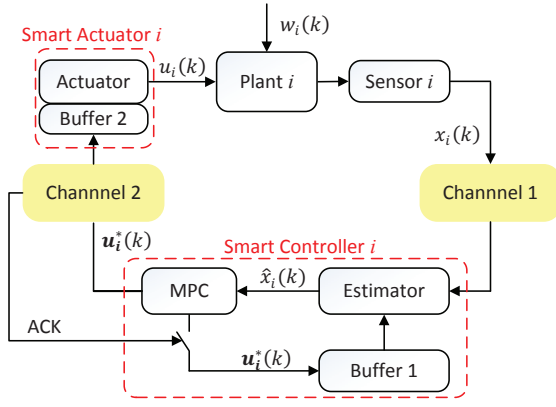


Fig. 2. The control architecture of packet-based MPC

(a) The buffer at the controller side is used to store the received control sequence of the actuator. This can be realized by the acknowledgement signal if the TCP-like protocol is applied. Let $\mathbf{b}_i(k)$ denote the content of the buffer at time k and $\mathbf{b}_i(0) = 0$, then we have

$$\begin{aligned} \mathbf{b}_i(k) &= d_{ai}(k)S\mathbf{b}_i(k-1) + (1-d_{ai}(k))\mathbf{u}_i(k) \\ \mathbf{u}_i(k) &= e_1^T \mathbf{b}_i(k) \end{aligned} \quad (2)$$

where $d_{ai}(k) = 1$ means the control sequence is received by the actuator of plant i , and $d_{ai}(k) = 0$ otherwise. $\mathbf{u}_i(k)$ is the control sequence and $u_i(k)$ is the actual control input. S and e_1 are defined via:

$$S \triangleq \begin{bmatrix} 0_p & I_p & 0_p & \cdots & 0_p \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0_p & \cdots & 0_p & I_p & 0_p \\ 0_p & \cdots & \cdots & 0_p & I_p \\ 0_p & \cdots & \cdots & \cdots & 0_p \end{bmatrix}, \quad e_1 \triangleq \begin{bmatrix} I_p \\ 0_p \\ \vdots \\ 0_p \end{bmatrix}$$

(b) The estimator estimates the current state of the plant according to whether the plant state is received. Specifically,

$$\hat{x}_i(k) = \begin{cases} x_i(k) & \text{if transmitted successfully} \\ \tilde{x}_i(k) & \text{otherwise} \end{cases} \quad (3)$$

where $\tilde{x}_i(k) = f_i(\hat{x}_i(k-1), u_i(k-1), 0)$ and $u_i(k-1)$ is defined in (2).

(c) The function of MPC is to calculate the control sequence $\mathbf{u}_i(k)$ at each time slot by solving a constrained optimization problem formulated as follows

$$\begin{aligned} \min_{\mathbf{u}_i(k)} & J_i(\hat{x}_i(k), \mathbf{u}_i(k)) \\ \text{s.t.} & \hat{x}_{i,j+1}(k) = f_i(\hat{x}_{i,j}(k), u_{i,j}(k), 0) \\ & \hat{x}_{i,0}(k) = \hat{x}_i(k) \\ & u_{i,j}(k) \in \mathbb{U}_i, \forall i = 1, \dots, n, j = 0, \dots, N-1. \end{aligned} \quad (4)$$

where $\hat{x}_i(k)$ is the estimated state of system i , N is the prediction horizon that need to be design, and MPC cost function of system i is $J_i(\hat{x}_i(k), \mathbf{u}_i(k)) = \sum_{j=0}^{N-1} l_i(\hat{x}_{i,j}(k), u_{i,j}(k)) + F_i(\hat{x}_{i,N}(k))$ with $l_i(\cdot)$ and $F_i(\cdot)$ being the stage cost and the terminal cost.

Smart Actuator The smart actuator is composed of an actuator and a buffer which is also used to store the received

control sequence and provide the control signal to actuator. The control input $u_i(k)$ provided by the buffer is defined also in (2).

B. Stability Analysis

First we give some necessary assumptions and then, based on which, the relationship between the stability and the packet transmission success probabilities are derived. Let q_s and q_c denote packet transmission success probability of sensor and controller, respectively. The stability results hold for any system i ($i = 1, 2, \dots, n$) under the following assumptions, so the subscript i is omitted for simplicity.

Assumption 2: [17] There exist constants $\lambda_x, \lambda_w, \lambda_l, \lambda_f$ and positive integer s such that for all x, y, u, w ,

$$\begin{aligned} \|f(x, u, w) - f(y, u, 0)\|^s &\leq \lambda_x \|x - y\|^s + \lambda_w d^s \\ |l(x, u) - l(y, u)| &\leq \lambda_l \|x - y\|^s \end{aligned} \quad (5)$$

$$|F(x) - F(y)| \leq \lambda_f \|x - y\|^s$$

Assumption 3: [17] The stage cost $l(\cdot)$ and terminal cost $F(\cdot)$ satisfy, for all x, u ,

$$\begin{aligned} l(x, u) &\geq \alpha_l \|x\|^s \\ F(x) &\geq \alpha_f \|x\|^s \end{aligned} \quad (6)$$

where α_l and α_f are two positive constants.

Assumption 4: [17] There exists a constrained control law $\kappa: \mathbb{R}^n \rightarrow \mathbb{U}$ such that

$$F(f(x, \kappa(x), 0)) + l(x, \kappa(x)) \leq F(x) \quad (7)$$

for all $x \in \mathbb{R}^n$.

Assumption 5: [18] There exist constant γ that satisfies $(1 - \min\{q_s, q_c\})\gamma < 1$, and $\eta \geq 0$ such that

$$F(f(x, 0, w)) \leq \gamma F(x) + \eta d^s \quad (8)$$

for all $x \in \mathbb{R}^n$ and $w \in \mathbb{R}^m$.

Similar to Lemma 5 in [17], the (8) is satisfied with $\gamma = \lambda_f \lambda_x / \alpha_f$ and $\eta = \lambda_f \lambda_x$ if $\max\{p_s, p_a\} \lambda_f \lambda_x / \alpha_f < 1$. Besides, The above Assumption implies that $V_f(f(x, 0, 0)) \leq \gamma V_f(x)$ holds for the nominal open-loop system.

Lemma 1: [18] For any system, suppose that Assumptions 2 ~ 5 hold, $(1-q_s)(1-q_c)\lambda_x < 1$, and the prediction horizon N is chosen such that

$$\left[\frac{(1-q_c)^{N+1} q_s}{1 - (1-q_c)\gamma} - \frac{(1-q_s)^{N+1} q_c}{1 - (1-q_s)\gamma} \right] \frac{\gamma - 1}{q_s - q_c} < \frac{\mu}{1 - \mu} \quad (9)$$

for $q_s \neq q_c$, and

$$\frac{N q_s (1 - \gamma + q_s \gamma) + (1 - (1 - q_s)^2 \gamma)}{(1 - \gamma + q_s \gamma)^2 / (\gamma - 1) (1 - q_s)^N} < \frac{\mu}{1 - \mu} \quad (10)$$

for $q_s = q_c$, where $\mu \triangleq \inf \frac{l(x)}{F(x)}$, then there exist constants c_1, c_2 and $0 < \rho < 1$ such that, for all $k \in \mathbb{N}_0$

$$\max_{\epsilon \in \{t_k, t_k+1, \dots, t_k+\Delta_k-1\}} E\{\|x(\epsilon)\|^s\} \leq c_1 (1 - \rho)^k \|x_0\|^s + c_2 d^s$$

where t_k is the k -th time instant that the actuator receives the control sequence, $\Delta_k \triangleq t_{k+1} - t_k$.

The above lemma reveals the relationship between the stability (boundedness) and packet transmission success probabilities. Given the prediction horizon N , q_s and q_c satisfies

the relevant probability requirements, the access strategy for each sensor and selection strategy for the central scheduler can be designed.

IV. TRANSMISSION STRATEGY AND GAME FORMULATION

In this part, the design objective for each system is described and some relevant transmission strategies are explained. Due to the decentralized architecture, the problem is formulated as a non-cooperative game. The definition of Nash equilibrium is then introduced.

A. The Objective of System

As indicated in the previous subsection, the requirements of the packet transmission success probabilities should be satisfied to guarantee the stability of the system. So the objective of each system can be stated as to minimize the average transmission power while meeting the requirements of the packet transmission success probabilities. Since q_s and q_c are correlated in these requirements and the co-design of which may be intractable, we have to design them separately.

Channel-aware Access Strategy for Sensor The access strategy may be random, i.e. the sensor accesses the channel with some positive probability. Let π_i denotes the access strategy of system i , then π_i decides whether to transmit or not during current time slot according to the available information, such as current CSI signal and possibly the access history. Let multi-strategy $\boldsymbol{\pi} = (\pi_1, \pi_2, \dots, \pi_n)$ represent the set of strategies of all systems and $\boldsymbol{\pi}_{-i} = (\pi_1, \pi_2, \dots, \pi_{i-1}, \pi_{i+1}, \dots, \pi_n)$ represents the set of strategies of all systems, except system i . For each system i , $p_{si}(\boldsymbol{\pi})$ denotes the access probability of the sensor, which depends not only on system's own strategy π_i but also the strategies of other systems $\boldsymbol{\pi}_{-i}$, and $q_{si}(\boldsymbol{\pi})$ is the packet transmission success probability. Therefore, for each system i , the objective is

$$\begin{aligned} & \min_{\pi_i} p_{si}(\pi_i, \boldsymbol{\pi}_{-i}) \\ \text{s.t. } & q_{si}(\pi_i, \boldsymbol{\pi}_{-i}) \geq \bar{q}_{si} \end{aligned}$$

where \bar{q}_{si} is the packet transmission success probability requirement satisfying $(1 - \bar{q}_{si})\gamma < 1$.

Due to the decentralized architecture where each system is self-optimizing, i.e. each system makes their own decision to fulfill its objective without using information of other systems. Despite of this, the decision can affect the performance of other systems through the collision channel, because the packet collision occurs if more than one systems transmit during the same time slot. So we adopt the non-cooperative game to analyze this behavior. In the following, we give the definition of Nash Equilibrium Point (NEP), where none of the systems can lower its access probability by unilaterally modifying its strategy π_i .

Definition 1: A multi-strategy $\boldsymbol{\pi} = (\pi_1, \pi_2, \dots, \pi_n)$ is a Nash Equilibrium Point if

$$\pi_i \in \arg \min_{\hat{\pi}_i} \{p_{si}(\hat{\pi}_i, \boldsymbol{\pi}_{-i}) : q_{si}(\hat{\pi}_i, \boldsymbol{\pi}_{-i}) \geq \bar{q}_{si}\}$$

In our problem setup, the decision whether to access the channel or not during one time slot only depend on the current CSI signal. Intuitively, it is low cost policy to increase the access probability when a better CSI signal is obtained. Therefore, in the following, we focus on the stationary access strategy and threshold strategy.

Definition 2: A stationary strategy is a mapping $\pi_i : \mathcal{H}_i \rightarrow [0, 1]$. Specifically, the explicit expression of a stationary strategy can be denoted by an m_i -dimensional vector $\mathbf{S}_i = [s_{i1}, s_{i2}, \dots, s_{im_i}] \in [0, 1]^{m_i}$, where s_{ij} is the access probability of system i when the channel condition is h_{ij} .

Definition 3: A threshold strategy is a special class of stationary strategy where $\mathbf{S}_i = [0, 0, \dots, s_{ir_i}, 1, \dots, 1]$ with $s_{ir_i} \in [0, 1]$.

For a threshold strategy, there exists some threshold h_{ir_i} , above which system i always transmit, below which it never transmit, and equal to which it transmit with probability s_{ir_i} . From [13], [19], we can conclude that in our problem, *an optimal stationary strategy is always a threshold strategy*. Further, there exists one-to-one relationship between the access strategy and the access probability, i.e., given the access probability p_{si} , the threshold strategy is determined and vice versa. Then, the CSI threshold and its corresponding transmission probability can be denoted by $h_{ir_i}(p_{si})$ and $s_{ir_i}(p_{si})$, where $s_{ir_i}(p_{si}) = \frac{p_{si} - \sum_{r=r_i+1}^{m_i} O(h_{ir})}{O_i(h_{ir_i})}$.

For sensor i , define the following function

$$\begin{aligned} H_i(p_{si}) &= s_{ir_i}(p_{si})O(h_{ir_i}(p_{si}))P(h_{ir_i}(p_{si})) \\ &+ \sum_{r=r_i+1}^{m_i} O(h_{ir})P(h_{ir}) \end{aligned} \quad (11)$$

This function is the packet transmission success probability of sensor i in the collision-free environment under the threshold strategy with access probability p_{si} . Therefore, the packet transmission success probability of sensor i is

$$q_{si}(p_{si}, \mathbf{p}_{s,-i}) = H_i(p_{si}) \prod_{j \neq i} (1 - p_{sj}) \quad (12)$$

Selection Strategy for Central Scheduler Suppose that the selection strategy is also random, i.e., the scheduler randomly choose a controller to access the channel during each time slot based on the given positive probability. For example, the probability of choosing controller i to access channel 2 is

$$p_{ci} = \frac{\bar{q}_{si}}{\sum_{j=1}^n \bar{q}_{sj}}, \quad (13)$$

and then the packet transmission success probability q_{ci} is

$$q_{ci} = \frac{\bar{q}_{si} \sum_{z_i \in \mathcal{H}_i} O(z_i)P(z_i)}{\sum_{j=1}^n \bar{q}_{sj}} \quad (14)$$

The probability requirement of the controller can be satisfied as long as $\sum_{z_i \in \mathcal{H}_i} O(z_i)P(z_i) \geq \sum_{j=1}^n \bar{q}_{sj}$. Actually, based on (14) and the fact that $(1 - \bar{q}_{si})\gamma_i < 1$, we have

$$(1 - q_{ci})\gamma_i < 1 + \bar{q}_{si}\gamma_i - \bar{q}_{si}\gamma_i \frac{\sum_{z_i \in \mathcal{H}_i} O(z_i)P(z_i)}{\sum_{j=1}^n \bar{q}_{sj}} < 1.$$

V. EQUILIBRIUM ANALYSIS AND ALGORITHM DESIGN

In this section, the existence and the characteristics of NEP are analyzed, and a decentralized channel access mechanism is designed for each sensor to achieve the NEP.

A. The Existence of Equilibrium Point

Lemma 2: The function $H_i(p_{si})$ is a continuous, strictly increasing and concave function with $H_i(0) = 0$.

Proof: From (11), it can be easily seen that $H_i(p_{si})$ is a piecewise-linear increasing function. The concavity is verified from the fact that the slope of H_i depends on $P(h_{ir_i}(p_{si}))$, and which decreases with $h_{ir_i}(p_{si})$ or p_{si} . $p_{si} = 0$ means no transmission, thus the transmission success probability is also 0. ■

Multi-strategy $\mathbf{p}_s = (p_{s1}, p_{s2}, \dots, p_{sn})$ is a probability representation of multi-strategy $\boldsymbol{\pi} = (\pi_1, \pi_2, \dots, \pi_n)$, then we have the following result about the NEP.

Theorem 1: A multi-strategy \mathbf{p}_s is an NEP if and only if it solves a set of equations

$$H_i(p_{si}) \prod_{j \neq i} (1 - p_{sj}) = \bar{q}_{si} \quad (15)$$

Proof: According to Definition 1, an NEP should satisfy

$$p_{si} = \min\{p_{si} : q_{si}(p_{si}, \mathbf{p}_{s,-i}) \geq \bar{q}_{si}\}$$

where $q_{si}(p_{si}, \mathbf{p}_{s,-i})$ is an increasing function of p_{si} . So the solution of the above minimization problem is also the solution of the equations (15). ■

Remark 1: The above theorem shows that the NEP also solves a set of equations named equilibrium equations. Further, this theorem is also a existence result of the NEP. That is, there exist an NEP (may be two NEPs) if and only if the equilibrium equations are feasible. As shown in [13], by dividing the requirement spaces of $\bar{\mathbf{q}}_s \in [0, 1]^n$ into two part: Ω and $\bar{\Omega}$, where the equilibrium equations are always feasible in Ω , and otherwise in $\bar{\Omega}$. There are two NEPs (one is strictly better than the other) for all $\bar{\mathbf{q}}_s$ in the interior of Ω , one in the upper boundary of Ω and none in $\bar{\Omega}$.

B. Channel Access Mechanism

In this part, we adopt a decentralized channel access mechanism based on the naive best-response dynamic that first proposed in [20] to adjust the access probability, then explain how the access probabilities converge to an NEP.

According to the equilibrium equations (15), the best response can be written as follows:

$$p_{si}(k+1) = H_i^{-1} \left(\frac{\bar{q}_{si}}{\prod_{j \neq i} (1 - p_{sj}(k))} \right) \quad (16)$$

where H_i^{-1} is the inverse function of the collision-free packet transmission success probability function. Although this inverse function always exist (Lemma 2), to obtain the explicit expression is intractable. Instead, we adopt the naive best-response (better response) here. In contrast to the best-response dynamic which is usually the optimal strategy of a user by giving the set of strategies of others, the main idea of the naive best-response (better response) is to find a strategy

(not necessarily the optimal ones) that has performance improved. Here, we update the transmission probability in proportion to the required increase of the packet transmission success probability. That is,

$$p_{si}(k+1) = \frac{\bar{q}_{si}}{q_{si}(p_{si}(k), \mathbf{p}_{s,-i}(k))} p_{si}(k) \quad (17)$$

Suppose that all users (sensors) update their access strategies repeatedly and $q_{si}(p_{si}, \mathbf{p}_{s,-i})$ can be perfectly estimated before each update. If all users start with low initial access probabilities and the equilibrium equations (15) are feasible, then it can be verified that with the above update policy, the multi-strategy \mathbf{p}_s is increased monotonically, i.e. $\mathbf{p}_s(0) < \mathbf{p}_s(1) < \dots < \mathbf{p}_s(k) < \dots$ and finally converge to the better NEP. The proofs of this conclusion follow similar lines in [20]. For the sake of brevity, the proofs are omitted.

Remark 2: For a practical implementation of the channel access mechanism (17), the packet transmission success probability $q_{si}(p_{si}, \mathbf{p}_{s,-i})$ is needed but unknown to sensor i . Thanks to the adopted TCP-like protocol, an approach to measure this probability is by monitoring the channel for some time slots, and counting the number of transmissions and successful transmissions by using acknowledgement signal sent from the controller. Note that the access strategies of other systems are not required when one sensor updates its transmission strategy. So the channel access mechanism can be implemented in decentralized manner.

VI. NUMERICAL SIMULATIONS

Consider the following three open-loop unstable systems:

$$x_i(k+1) = x_i(k) + a_i \sin(x_i(k)/5) + u_i(k) + w_i \quad (18)$$

where $w_i \in [-0.1, 0.1]$, $i = 1, 2, 3$ and $a_1 = 0.25$, $a_2 = 0.6$, $a_3 = 0.4$. Suppose that the control input has the constraint $-1 \leq u_i \leq 1$. The initial states $x_1(0) = 10$, $x_2(0) = 10$ and $x_3(1200) = -10$ with accessing time instants 0, 0 and 1200, respectively. The stage cost function and the terminal cost function are chosen as $l_i(x) = \|x\|$ and $F_i(x) = 2\|x\|$, thus Assumption 2 and 3 can be satisfied with $\alpha_{Fi} = \lambda_{Fi} = 2$, $\lambda_{wi} = \lambda_{li} = \alpha_{li} = s = 1$ and $\lambda_{xi} = 1 + a_i/5$. Assumption 5 holds with $\gamma_i = 1 + a_i/5$ and $\eta_i = 1$. The Assumption 4 is satisfied with the terminal controller $\kappa_i(x) = -a_i \sin(x/5) - 0.05x$. For the channel model, we assume that the CSI is classified by $\mathcal{H}_1 = \mathcal{H}_2 = \mathcal{H}_3 = \{0.5, 1.5, 2.0, 2.5\}$ with the probability distribution $O_i(h) = [0.05, 0.2, 0.65, 0.1]$ and transmission power of each sensor is $\xi_i = 20mW$. If a specific error-correcting code is applied, the successful decoding probability is nearly $P_i(h) = [0.1, 0.9, 0.96, 0.99]$. We have $\sum_{j=1}^4 O_i(h_j)P_i(h_j) = 0.902$. By letting $N_1 = 16$, $N_2 = N_3 = 14$, $\bar{q}_{s1} = 0.07$, $\bar{q}_{s2} = 0.14$, $\bar{q}_{s3} = 0.10$ and adopting the selection strategy (13), the probability requirements of stability are satisfied. The channel access mechanism (17) with initial transmission probabilities $p_{s1}(0) = p_{s2}(0) = p_{s3}(1200) = 0.05$ is performed every 100 time slots. The packet-based MPC is applied, then the state responses of these systems are shown in Fig.3. Fig.4 shows the access probability evolutions

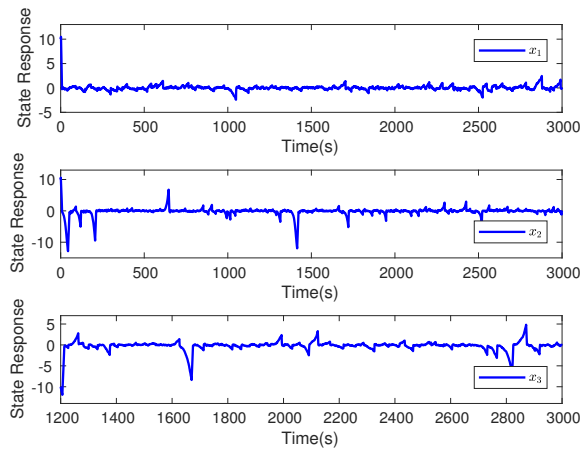


Fig. 3. State responses of all closed-loop systems

of each sensor. We observe that the access probabilities converge to the NPE fast. Before the third system accessing the channel, the access probabilities of two systems converge to an equilibrium with $p_{s1} = 0.0838$, $p_{s2} = 0.1560$, and the corresponding threshold strategies are $s_1 = [0, 0, 0, 0.8380]$ and $s_2 = [0, 0, 0.0862, 1]$. After the introduction of the new system, the access probabilities update and converge to a new equilibrium $\mathbf{p}_s = [0.1008, 0.1853, 0.1391]$ with the corresponding threshold strategies $s_1 = [0, 0, 0.0012, 1]$, $s_2 = [0, 0, 0.1312, 1]$, $s_3 = [0, 0, 0.0602, 1]$, respectively.

VII. CONCLUSION

In this paper, we have considered the scheduling and control for WCS with multiple systems communicating over a shared collision channel. The packet-based model predictive controller, the channel-aware access strategies for sensors and the selection strategy for scheduler have been designed. The characteristics of the NPE has been analyzed by using the non-cooperative game theory, and a decentralized channel access mechanism is designed to achieve the NPE. However, the selection strategy of the scheduler ignores the CSI and is too simple, so a more-refined strategy need to be further investigated. Future research also includes the study of the multi-channel case [8], [19] and the state-based access strategy [21].

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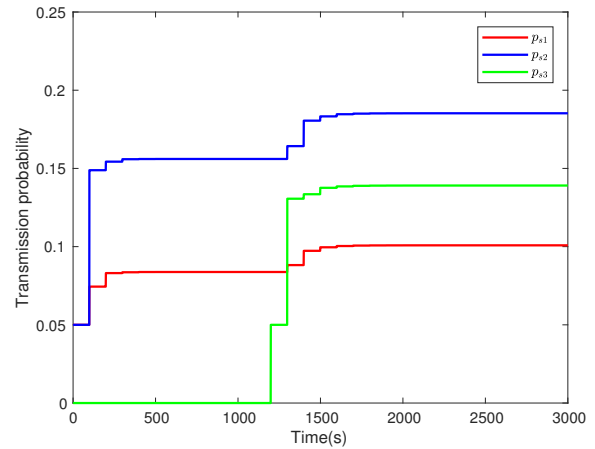


Fig. 4. Update of access probability of each sensor

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