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Robust Approximation-Based Event-Triggered MPC for Constrained Sampled-Data Systems*

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Abstract In this paper, an approximation-based event-triggered model predictive control (AETMPC) strategy is proposed to implement event-triggered model predictive control for continuous-time constrained nonlinear systems under the digital platform. In the AETMPC strategy, both of the optimal control problem (OCP) and the triggering conditions are defined in a discrete-time manner based on approximate discrete-time models, while the plant under control is continuous time. In doing so, sensing load is alleviated because the triggering condition does not need to be checked continuously, and the computation of the OCP is simpler since which is calculated in the discrete-time framework. Meanwhile, robust constraints are satisfied in a continuous-time sense by taking inter-sampling behavior into

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consideration, and a novel constraint tightening approach is presented accordingly. Furthermore, the feasibility of the AETMPC strategy is analyzed and the associated stability of the overall system is established. Finally, this strategy is validated by a numerical example.

Key words Approximations, event-triggered control, model predictive control, sampled-data systems.

1 Introduction

Networked control systems (NCSs) have been widely used in recent years due to the fact that it has many benefits which include but are not limited to simpler maintenance, installation, and lager flexibility[1]. However, the imperfect communication networks also bring some challenges. For example, the limited communication resources (e.g., the limited channel capacity, the battery-powered sensors in wireless networks) imposes the needs of efficient transmissions for NCSs. Event-triggered control is a relatively satisfactory scheme because the sensing or control data is transmitted only when some prescribed conditions are violated. In this way, event-triggered control can save the network resources. Therefore, studies on event-triggered control for NCSs have arisen great attention, see, e.g., [2–7]. In another research line, model predictive control (MPC) is a powerful tool to cope with constrained nonlinear systems, and thus has been widely utilized to study nonlinear NCSs[8]. But performing MPC needs to compute an optimal control problem (OCP) which in general causes a large computation burden. Fortunately, event-triggered control mentioned above provides an opportunity to alleviate this issue through reducing the number of solving the OCP. By recognizing the aforementioned advantages, we combine the event-triggered mechanism (transmission policy) and MPC (control scheme), and an efficient event-triggered MPC (ETMPC) algorithm can then be obtained.

In this work, we focus on how to implement the ETMPC algorithms for the nonlinear NCSs with continuous-time plant and digital controller. One may notice that both the continuous- and discrete-time signals co-exist in overall systems, which can be named "sampled-data systems". That is, the ETMPC for such system is also called sampled-data ETMPC.

Generally speaking, studies on sampled-data ETMPC can be mainly divided into two categories, namely, continuous-time ETMPC[2–4] and discrete-time ETMPC[5–7]. The continuous-time ETMPC is more natural as the considered plants are modeled by differential equations subjected to continuous-time constraints. Hence, the corresponding solutions lead to the constraints satisfaction in a continuous-time sense. But for the event-triggered mechanism, the prescribed triggering conditions need to be checked continuously repeated, which results in the promotion of the sensing cost[2, 3]. To overcome this drawback, ETMPC for continuous-time nonlinear systems with intermittent sampling is proposed in [4], where the triggering condition is checked only at specified sampling instants. However, in all these works, the differential equations are regarded as a constraint in the OCP, which is computationally intractable in solving such an OCP[9]. On the contrary, in discrete-time ETMPC, the computation of the OCP is tractable by exploiting the discrete-time model (DTM) delineated by difference equations directly and the triggering condition does not need to be continuously evaluated. Therefore, the implementation of ETMPC in a discrete-time framework is simpler and has lower sensing load[7]. Nonetheless, two problems also arise: 1) The required DTM may not be available for generic continuous-time nonlinear systems; 2) The state constraints are considered only at each sampling instant, thus the inter-sampling behavior is neglected, leading to continuous-time state constraints unsatisfaction[10].

Considering the above two frameworks with their advantages and disadvantages, we propose an approximation-based ETMPC (AETMPC) strategy, which introduces approximate DTM[11] to approximate the original continuous-time dynamics. With this strategy, the advantages of the continuous-time constraints satisfaction in the continuous-time framework and the simple computation as well as lower sensing load in the discrete-time framework are all maintained. Additionally, the recursive feasibility and stability are established. The main contributions of this paper are twofold:

• The advantages of our proposed AETMPC strategy over the existing results in [2–4, 12] have three aspects. Firstly, the approximate DTM and the discretized cost function in our AETMPC facilitates the solution of OCP. Secondly, the event-triggered condition is checked periodically (rather than continuously repeated), which results in the reduction of network load and energy consumption of the sensor. Thirdly, the event-triggered conditions provide a guideline to determine the allowable sampling period.

• A novel constraint tightening approach for the formulation of OCP is developed. Compared with the earlier studies on ETMPC[5–7] where the constraints are either neglected or considered only at sampling instant, our approach accounts fully for the inter-sampling behavior (the states during two consecutive sampling instants), ensuring constraints satisfaction in a continuous-time sense in presence of model error and external disturbances.

This paper is organized as follows. In Section 2, the problem statement is provided. Section 3 proposes the approximation-based event-triggered model predictive control strategy. Section 4 performs the feasibility and stability analysis. The effectiveness of our proposed strategy is verified by a numerical example in Section 5. Section 6 summarizes this paper.

Notations Let $\mathbb{R}$ and $\mathbb{Z}_0$ represent the real and nonnegative integers, respectively. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. A matrix P is called positive definite if $P > 0$. For a vector x, its Euclidean norm is denoted by $\|x\| := \sqrt{x^{\mathrm{T}}x}$, and the P-weighted norm is denoted by $\|x\|_P := \sqrt{x^{\mathrm{T}}Px}$. $\boldsymbol{w}_{[t_1,t_2]}$ is a signal from t_1 to t_2 and its subscript can be omitted for simplify when it can be derived from context. $x(t_{k+i}|t_k)$, $k, i \in \mathbb{Z}_0$, indicates the ith predicted state based on the current state $x(t_k)$, and $u(t_{k+i}|t_k)$ is similar. For two sets $A, B \subseteq \mathbb{R}^n$, $A \ominus B := \{x : x + y \in A, \forall y \in B\}$ denotes the Pontryagin difference set. $\mathcal{K}$ denotes a class-$\mathcal{K}$, $\mathcal{K}_\infty$ is a class-$\mathcal{K}_\infty$, and $\mathcal{KL}$ is a class-$\mathcal{KL}$ function, see [13] for details.

2 Problem Statement

Consider the following continuous-time constrained nonlinear control system

$$\dot{x}(t) = f(x(t), u(t)) + w(t), \quad t \geq 0, \tag{1}$$

where $x(t) \in \mathbb{R}^n$ represents the system state, $u(t) \in \mathbb{R}^m$ represents the control input, and $w(t) \in \mathbb{R}^n$ is the external disturbance. They are subject to continuous-time constraints described below

$$x(t) \in \mathcal{X}, \quad u(t) \in \mathcal{U}, \quad w(t) \in \mathcal{W}, \tag{2}$$

where the compact sets $\mathcal{X},\mathcal{U}$ contain the origin, and $\mathcal{W} = \{w(t) \in \mathbb{R}^n : \|w\|_R \leq \xi\}$ with a positive definite matrix R. Let $\varphi(t; x_0, \boldsymbol{u}, \boldsymbol{w})$ be the solution of the system (1) at time t with initial value $x(0) = x_0$, control input $\boldsymbol{u}$, and disturbance $\boldsymbol{w}$. In addition, the function f should satisfy the following assumption.

Assumption 2.1 $f(x, u)$ is local Lipschitz continuous with constant $L_f > 0$ depending on the weighted matrix R, i.e., $\|f(x, u) - f(y, u)\|_R \leq L_f \|x - y\|_R$, $\forall x, y \in \mathcal{X}, u \in \mathcal{U}$.

Let $t_k = kT$ denote the sampling instant with $k \in \mathbb{Z}_0$ and T being the sampling period, and let t_{k_j} be the jth triggering time instant. The schematic block diagram of the overall system structure is depicted in Figure 1, and the whole executing procedure of the sampled-data ETMPC algorithm is elaborated as the following steps:

Step 1 At any sampling instants t_k, the sensor measures the state $x(t_k)$, and the event trigger checks the triggering conditions based on $x(t_k)$. If the triggering conditions are transgressed, $x(t_k)$ is transmitted to the remote controller and denote the current triggering instant by $t_{k_j} = t_k$; otherwise, perform Step 3.

Step 2 The remote controller (MPC algorithm with prediction horizon $t_{k_j+N} - t_{k_j}$ and initial condition $x(t_{k_j})$) is carried out to generate the predictive control input sequence $\widehat{\boldsymbol{u}}(t_{k_j}) = \{\widehat{u}(t_{k_j}|t_{k_j}), \widehat{u}(t_{k_j+1}|t_{k_j}), \cdots, \widehat{u}(t_{k_j+N}|t_{k_j})\}$.

Step 3 The actuator implemented with a zero-order holder applies the control input $u(t) = \widehat{u}(t_k|t_{k_j}), \forall t \in [t_k, t_{k+1})$ to the continuous-time plant, where t_{k_j} is the latest triggering instant satisfying $t_{k_j} \leq t_k < t_{k_{j+1}}$.

The above three steps describe the basic mechanism of the sampled-data ETMPC and the roles of all components in Figure 1. What remains to be designed are the MPC algorithm and the triggering condition.

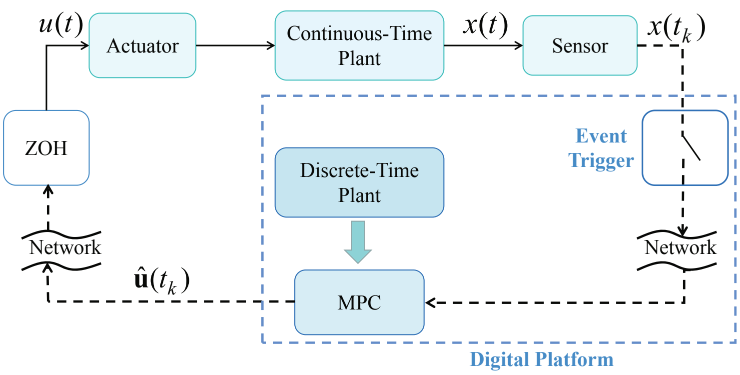

Figure 1 The framework of sampled-data ETMPC (The solid line and the dashed line denote the continuous- and discrete-time signal, respectively)

At each triggering instant t_{k_j}, performing the MPC algorithm means an OCP is solved to provide the optimal predictive control sequence $\widehat{\boldsymbol{u}}(t_{k_j})$. Therefore, the formulation of the OCP (including the cost function and constraints) is the key to design the sampled-data ETMPC algorithm successfully. For continuous-time plant, the cost function based on the state $x(t_{k_j})$ in general has the following expression (see, e.g., [3, 12, 14])

$$J(x(t_{k_j}), \boldsymbol{u}(t_{k_j}), N) = \int_{t_{k_j}}^{t_{k_j+N}} l(\widehat{x}(\tau|t_{k_j}), \widehat{u}(\tau|t_{k_j}))d\tau + g(\widehat{x}(t_{k_j+N}|t_{k_j})), \tag{3}$$

where $l(x,u) : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^+$ is the stage cost and $g(x) : \mathbb{R}^n \to \mathbb{R}^+$ is the terminal cost. $\widehat{x}(t|t_k)$ and $\widehat{u}(t|t_k)$ are, respectively, the predicted state and control input. Both of them are subject to constraints in (2) and are calculated depending on the DTM of the original system (1)

$$\begin{aligned} &\widehat{x}(t_{k+i+1}|t_k) = F_T^e(\widehat{x}(t_{k+i}|t_k), \widehat{u}(t_{k+i}|t_k)),\\ &\widehat{x}(t_k|t_k) = x(t_k),\\ &\widehat{x}(t_{k+i}|t_k) \in \mathcal{X}, \quad \widehat{u}(t_{k+i}|t_k) \in \mathcal{U},\\ &t_{k+i} = t_k + iT, \end{aligned} \tag{4}$$

where F_T^e is the disturbance-free exact DTM, while the exact DTM of the system (1) is expressed as $x(t_{k+1}) = F_T^e(x(t_k), u(t_k)) + w_{T,t_k}$ with $F_T^e(x(t_k), u(t_k)) = x(t_k) + \int_{t_k}^{t_{k+1}} f(x(\tau), u(t_k))d\tau$ and $w_{T,t_k} := \int_{t_k}^{t_{k+1}} w(\tau)d\tau$.

If we combine the cost function (3) and the constraints (4) to formulate the OCP, some problems arise: (i) The cost function requires continuous-time $\widehat{x}(\tau|t_k)$ and $\widehat{u}(\tau|t_k)$, which are not available under the DTM in (4); (ii) The explicit expression of the exact DTM F_T^e cannot be obtained for general nonlinear systems; (iii) The continuous-time constraints (2) may not be satisfied as the inter-sampling behavior is neglected in constraints (4).

After formulating the MPC algorithm, we then need to give the triggering condition that determines the time instants when the state is transmitted and the MPC algorithm is performed. The triggering condition is usually designed as

$$\|x(t_k) - \widehat{x}(t_k|t_{k_j})\| \le \eta, \tag{5}$$

where $x(t_k)$ is the actual state, $\widehat{x}(t_k|t_{k_j})$ is the predicted one, and η is the triggering threshold. Note that the triggering threshold not only reflects the allowable prediction error of the state but also affects the feasibility of the sampled-data ETMPC algorithm[4].

With the above descriptions, our objective is to reformulate the OCP (the cost function and constraints) to overcome the problems caused by (3) and (4), and to design the triggering threshold η in (5) such that the sampled-data ETMPC algorithm is feasible and the overall system can be stabilized.

Before proceeding, we introduce a necessary definition.

Definition 2.1 (see [15]) Given two compact sets $\mathcal{X}, \mathcal{W} \subseteq \mathbb{R}^n$ that contain the origin. A control system $\dot{x}(t) = f(x(t), w(t))$ is input-to-state practically stable (ISpS) if there exist functions $\beta \in \mathcal{KL}$, $\gamma \in \mathcal{K}_\infty$ and a constant $\sigma > 0$ such that, for any initial condition $x_0 \in \mathcal{X}$ and each bounded input $w(t)$, the solution $x(t)$ exists for all $t \geq 0$ and satisfies, with $\xi = \sup_{w\in\mathcal{W}} \|w\|$,

$$\|x(t)\| \leq \beta(\|x_0\|, t) + \gamma(\xi) + \sigma. \tag{6}$$

3 AETMPC Strategy

In this section, to achieve our objective, the AETMPC strategy is proposed. First of all, the redefined OCP is introduced in Subsection 3.1, based on which the event-triggered scheme is designed in Subsection 3.2.

3.1 The OCP in the AETMPC

This part reformulates the OCP in the AETMPC on the basis of the approximate approach. The key is to redefine the cost function and the predictive model. First, we introduce the approximate DTM which is significant in solving the OCP. Given the sampling period T, the disturbance-free approximate DTM of the system (1) is denoted by

$$x(t_{k+1}) = F_T^a(x(t_k), u(t_k)), \tag{7}$$

where the explicit expression of F_T^a depends on the adopted numerical methods. Taking the Euler approximation for example, we have $F_T^a(x(t_k), u(t_k)) = F_T^{\mathrm{Euler}}(x(t_k), u(t_k)) := x(t_k) + Tf(x(t_k), u(t_k))$. Furthermore, F_T^a should satisfy the following assumptions.

Assumption 3.1 Given any sampling period $T > 0$, F_T^a is continuous in u, and the following two inequalities

$$\|F_T^a(x_1, u) - F_T^a(x_2, u)\|_R \leq \mathrm{e}^{L_f T}\|x_1 - x_2\|_R, \tag{8}$$

$$\|F_T^e(x_1, u) - F_T^a(x_1, u)\|_R \leq T\varrho(T) \tag{9}$$

hold with a $\mathcal{K}_\infty$ function ϱ for all $x_1, x_2 \in \mathcal{X}$ and $u \in \mathcal{U}$.

Remark 3.1 Assumptions 2.1 and 3.1 are standard for the exact and the approximate DTM, more details can be found in [11, 16]. The inequality (8) implies that F_T^a satisfies a local Lipschitz condition. The inequality (9) limits the one-step model error between F_T^e and F_T^a over the time interval $[t_k, t_{k+1}]$, and the more accurate F_T^a is, the smaller one-step model error will be. It is necessary to emphasize that the inequality (9) can be checked even though the explicit expression of F_T^e is not available.

Based on Assumption 3.1, we first give the state error between the original system (1) and the approximate DTM (7) for defining the following tightened set.

Lemma 3.2 *Suppose that Assumptions* 2.1 *and* 3.1 *hold. Then given* $x(t_{k+i}) \in \mathcal{X}$, $\boldsymbol{u}_{[t_{k+i}, t_{k+i}+\tau]} \in \mathcal{U}$, *the state error* $\boldsymbol{e}_{[t_{k+i}, t_{k+i}+\tau]} := \varphi(t_{k+i}+\tau; x(t_{k+i}), \boldsymbol{u}_{[t_{k+i}, t_{k+i}+\tau]}, \boldsymbol{w}_{[t_{k+i}, t_{k+i}+\tau]}) - \widehat{x}(t_{k+i}|t_k)$ *is bounded by*

$$\|\boldsymbol{e}_{[t_{k+i}, t_{k+i}+\tau]}\|_R \leq \mu\tau + \frac{\mathrm{e}^{iL_f T} - 1}{\mathrm{e}^{L_f T} - 1}(T\varrho(T) + \xi T), \tag{10}$$

where $\tau \in [0, T)$ *and* μ *is a constant that makes system function in* (1) *bounded from above, i.e.,* $\|f(x(t), u(t)) + w(t)\|_R \le \mu$, $\forall x(t) \in \mathcal{X}, u(t) \in \mathcal{U}$, *and* $w(t) \in \mathcal{W}$.

Proof Note that the constant μ always exists due to the compactness of $\mathcal{X}, \mathcal{U}$ and $\mathcal{W}$. First of all, the state error between the exact DTM and the approximate one is $\|F_T^e(x(t_k), u(t_k)) + w_{T,t_k} - F_T^a(x(t_k), u(t_k))\|_R \le T\varrho(T) + \xi T$. Then, we cauculate $e(t_{k+i}) = x(t_{k+i}) - \widehat{x}(t_{k+i}|t_k)$. According to Assumption 3.1, the fact $\widehat{x}(t_k|t_k) = x(t_k)$ and $\widehat{u}(t_k|t_k) = u(t_k)$, and the triangle inequality, one obtains $e(t_{k+1}) \le T\varrho(T) + \xi T + \mathrm{e}^{L_f T}\|\widehat{x}(t_k|t_k) - x(t_k)\|_R = T\varrho(T) + \xi T$. Therefore, by induction, we have

$$\begin{aligned} \|e(t_{k+i})\|_R &= T\varrho(T) + \xi T + \mathrm{e}^{L_f T}\|x(t_{k+i-1}) - \widehat{x}(t_{k+i-1}|t_k)\|_R \\ &\le \frac{\mathrm{e}^{iL_f T} - 1}{\mathrm{e}^{L_f T} - 1}(T\varrho(T) + \xi T). \end{aligned} \tag{11}$$

Finally, considering the triangle inequality, it follows that

$$\begin{aligned} \|\boldsymbol{e}_{[t_{k+i}, t_{k+i}+\tau]}\|_R &\le \|\varphi(t_{k+i} + \tau; x(t_{k+i}), \boldsymbol{u}_{[t_{k+i}, t_{k+i}+\tau]}, \boldsymbol{w}_{[t_{k+i}, t_{k+i}+\tau]}) - x(t_{k+i})\|_R + \|e(t_{k+i})\|_R \\ &\le \mu\tau + \frac{\mathrm{e}^{iL_f T} - 1}{\mathrm{e}^{L_f T} - 1}(T\varrho(T) + \xi T). \end{aligned} \tag{12}$$

This proof is completed. ∎

On the basis of Lemma 3.2, the tightened set is defined as $\mathcal{X}_i = \mathcal{X} \ominus \mathcal{B}_i$, where $\mathcal{B}_i := \{x \in \mathbb{R}^n : \|x\|_R \le \mu\tau + \frac{\mathrm{e}^{iL_f T} - 1}{\mathrm{e}^{L_f T} - 1}(T\varrho(T) + \xi T)\}$. One can notice that if the predicted state satisfies the pointwise constraint $\widehat{x}(t_{k+i}|t_k) \in \mathcal{X}_i$, then the continuous-time state constraint (2) is satisfied, i.e., $\varphi(t_{k+i} + \tau; x(t_{k+i}), \boldsymbol{u}, \boldsymbol{w}) \in \mathcal{X}$.

For simplicity, let $\widehat{x}_{k+i|k} = \widehat{x}(t_{k+i}|t_k)$ and $\widehat{u}_{k+i|k} = \widehat{u}(t_{k+i}|t_k)$. With the above preliminaries, the OCP at time t_k in the AETMPC can be stated as follows:

$$\begin{aligned} \min_{\boldsymbol{u}} \ & J_T(x(t_k), \boldsymbol{u}(t_k), N) \\ \text{s.t.} \ & \widehat{x}_{k+i+1|k} = F_T^a(\widehat{x}_{k+i|k}, \widehat{u}_{k+i|k}), \\ & \widehat{x}_{k+i|k} \in \mathcal{X}_i, \\ & \widehat{u}_{k+i|k} \in \mathcal{U}, \\ & \widehat{x}_{k+N|k} \in \mathcal{X}_f \end{aligned} \tag{13}$$

with $i = 0, 1, \cdots, N-1$, $\widehat{x}_{k|k} = x_k$ and the T-related cost function being given as

$$J_T(x(t_k), \boldsymbol{u}(t_k), N) = \sum_{i=0}^{N-1} Tl(\widehat{x}(t_{k+i}|t_k), \widehat{u}(t_{k+i}|t_k)) + g(\widehat{x}(t_{k+N}|t_k)), \tag{14}$$

where $l(x, u) = \|x\|_Q^2 + \|u\|_P^2$ is the stage cost, $g(x) = \|x\|_R^2$ is the terminal cost with Q, P, R being positive definite matrices, and $\mathcal{X}_f = \{\widehat{x} : \|\widehat{x}\|_R \le \varepsilon_f\}$ is the terminal state constraint set. Let $\widehat{\boldsymbol{u}}(t_k) = (\widehat{u}_{k|k}, \widehat{u}_{k+1|k}, \cdots, \widehat{u}_{k+N|k})$ be the optimal solution of the OCP at time t_k and $\widehat{\boldsymbol{x}}(t_k) = (\widehat{x}_{k|k}, \widehat{x}_{k+1|k}, \cdots, \widehat{x}_{k+N|k})$ be the corresponding optimal state sequence. The optimal value function at time t_k is $J_T^0(x_k) = J_T(x(t_k), \widehat{\boldsymbol{u}}(t_k), N)$.

Remark 3.3 By using the approximate DTM F_T^a (7) and the redefined T-related cost function (14), the above two difficulties (i) and (ii) in Section 2 are overcome. Note that the T-related cost function chosen here is to ensure the stability of the exact DTM, which is similar to [16]. Additionally, the novel tightened set guarantees the continuous-time constraints (2) satisfaction, solving the difficulty (iii).

With the above preliminaries, the MPC algorithm can be computationally tractable under the digital platform. The following part aims at designing the event-triggered scheme.

Some assumptions needed to develop the main results are given here.

Assumption 3.2 The terminal state constraint set $\mathcal{X}_f$, the terminal cost $g(x)$, the auxiliary control law $h(x)$, the stage cost function $l(x,u)$, and another important set $\varXi$ satisfy the following properties:

1) $0 \in \mathcal{X}_f$, $\mathcal{X}_f \subset \varXi = \{\widehat{x} : \|\widehat{x}\|_R \le \varepsilon_\varXi\}$ with $0 < \varepsilon_f < \varepsilon_\varXi$ and $\varXi \subseteq \{x \in \mathcal{X}_{N-1} : h(x) \in \mathcal{U}\}$;

2) $F_T^a(x, h(x)) \in \mathcal{X}_f$, $\forall x \in \varXi$;

3) $g(F_T^a(x, h(x))) - g(x) \le -Tl(x, h(x))$, $\forall x \in \varXi$;

4) $g(x)$ is Lipschitz continuous in $\varXi$ with a constant $L_g > 0$ relying on weighted matrix R, i.e., $|g(x) - g(y)| \le L_g\|x - y\|_R$, $\forall x, y \in \varXi$;

5) $g(x)$ is bounded, that is, $\widetilde{\alpha}_1(\|x\|) \le g(x) \le \widetilde{\alpha}_2(\|x\|)$ with $\widetilde{\alpha}_1, \widetilde{\alpha}_2 \in \mathcal{K}_\infty$, $\forall x \in \varXi$;

6) $l(x,u)$ is Lipschitz continuous with a constant L_l depending on weighted matrix R, i.e., the inequality $|l(x,u) - l(y,u)| \le L_l\|x - y\|_R$, $\forall x, y \in \mathcal{X}$, $u \in \mathcal{U}$ holds;

7) $l(x,u) \ge \widetilde{\alpha}_3(\|x\|)$ with $\widetilde{\alpha}_3 \in \mathcal{K}_\infty$, $\forall x \in \mathcal{X}$, $u \in \mathcal{U}$;

8) there exist $\theta \in \mathcal{K}_\infty$ such that $f(x,u) \le \max\{\varepsilon, \theta(1/\varepsilon)l(x,u)\}$ holds for all $x \in \mathcal{X}$, $u \in \mathcal{U}$ and $\varepsilon > 0$.

Remark 3.4 The properties 1)–7) are fairy standard in the literature of MPC, which also provide some guidelines to choose $h(x)$ $\mathcal{X}_f$ and $\varXi$, several methods can be found in [17, 18]. The property 8) guarantees that the solution of the sampled-data system is uniformly bounded over T, which is important to guarantee that the overall system is stable, and it can be easily satisfied for generic nonlinear systems, see [19, Section 4.3] for more details.

3.2 Event-Triggered Scheme

This part specifies the triggering condition, i.e., determines the triggering threshold of (5). Recalling the triggering condition (5), we here specify it as follows

$$\begin{aligned} &\|x_k - \widehat{x}_{k|k_j}\|_R \le \eta, \\ &k - k_j \le N, \end{aligned} \tag{15}$$

where $\eta = \frac{\varepsilon_\varXi - \varepsilon_f}{\mathrm{e}^{NL_fT}} - \frac{T\varrho(T) + \xi T}{\mathrm{e}^{L_fT}}$, denoted as the triggering threshold. When either of (15) is violated, the next triggering instant is set as $t_{k_{j+1}} = t_k$. In addition, to ensure the validity of the triggering condition, i.e., the triggering threshold should be $\eta \geq 0$. As a result, the model error and the external disturbance should satisfy

$$T\varrho(T) + \xi T \leq \frac{\varepsilon_{\Xi} - \varepsilon_f}{\mathrm{e}^{(N-1)L_f T}}. \tag{16}$$

Remark 3.5 1) The main idea of determining the triggering threshold η is to ensure the feasibility, which is elaborated in Theorem 4.2. Further note that the above triggering condition (15) involves the model error $T\varrho(T)$. It can be derived from (15) that a smaller model error $T\varrho(T)$ leads to a higher threshold, which implies a larger triggering interval. This leads to the reduction of the energy resource further. However, a more accurate approximate model (with smaller model error) always has a more complex expression, which also increases the difficulty of ETMPC algorithms design. The associated examples will be illustrated in Section 5.

2) The inequality (16) also provides a guideline to determine the allowable sampling period T once we know the upper bound of external disturbance ξ in practice. For example, if we know that the upper bound of a disturbance is ξ_1, we need to choose a T_1 such that $T_1\varrho(T_1) + \xi_1 T_1 \leq (\varepsilon_{\Xi} - \varepsilon_f)/\mathrm{e}^{NL_f T_1}$ holds instead of choosing randomly.

So far, the reformulation of the OCP and the determination of the triggering condition are achieved, based on which the sampled-data ETMPC algorithm for the system (1) is implemented by our AETMPC strategy. We then verify the validity of the AETMPC strategy.

4 Analysis of the AETMPC Strategy

This section analyzes the feasibility of our AETMPC strategy and the stability of the overall system.

4.1 Feasibility Analysis

The following lemma is utilized for the feasibility establishment.

Lemma 4.1 *If $x \in \mathcal{B}_{i+m}$ and $y \in \mathbb{R}^n$ satisfies*

$$\|x - y\|_R \leq \mathrm{e}^{iL_f T} \frac{\mathrm{e}^{mL_f T - 1}}{\mathrm{e}^{L_f T} - 1}(T\varrho(T) + \xi T), \tag{17}$$

then $y \in \mathcal{B}_i$.

Proof Let $z = y - x + e_i$ with $e_i \in \mathcal{B}_i$, it follows that

$$\begin{aligned}
\|z\|_R &\leq \|y - x\|_R + \|e\|_R \\
&\leq \left(\mathrm{e}^{iL_f T}\frac{\mathrm{e}^{mL_f T} - 1}{\mathrm{e}^{L_f T} - 1} + \frac{\mathrm{e}^{iL_f T} - 1}{\mathrm{e}^{L_f T} - 1}\right)(T\varrho(T) + \xi T) \\
&= \frac{\mathrm{e}^{(i+m)L_f T} - 1}{\mathrm{e}^{L_f T} - 1}(T\varrho(T) + \xi T).
\end{aligned}$$

Thus, $z \in \mathcal{B}_{i+m}$. Noting that $y + e_i = z + x \in \mathcal{X}$, we can conclude that $y \in \mathcal{B}_i$. ∎

Theorem 4.2 *Consider the system* (1) *with the event-triggered condition* (15). *Assume that the model error and the external disturbance are subject to* (16), *and Assumptions* 2.1 *and*3.1 *hold, then the AETMPC strategy is feasible.*

Proof Suppose that the solution of the OCP at t_{k_j} is $\widehat{\boldsymbol{u}}(t_{k_j}) = (\widehat{u}_{k_j|k_j}, \widehat{u}_{k_j+1|k_j}, \cdots, \widehat{u}_{k_j+N|k_j})$. Based on $\widehat{\boldsymbol{u}}(t_{k_j})$, we construct a feasible solution $\overline{\boldsymbol{u}}(t_{k_{j+1}})$ at time $t_{k_{j+1}}$ to prove the feasibility of the AETMPC, as follows:

$$\overline{u}_{k_{j+1}+i|k_{j+1}} = \begin{cases} \widehat{u}_{k_{j+1}+i|k_j}, & i = 0, 1, \cdots, N-1-(k_{j+1}-k_j), \\ h(\overline{x}_{k_{j+1}+i|k_{j+1}}), & i = N-(k_{j+1}-k_j), N-(k_{j+1}-k_j)+1, \cdots, N-1. \end{cases} \tag{18}$$

At first, we derive an upper bound of the estimated error at time $t_{k_{j+1}}$ based on the above triggering condition (15), which is a prerequisite for the feasibility. By virtue of the inequality (11), we obtain

$$\|x_{k_{j+1}} - \widehat{x}_{k_{j+1}|k_j}\|_R \leq T\varrho(T) + \xi T + \mathrm{e}^{L_f T}\|x_{k_{j+1}-1} - \widehat{x}_{k_{j+1}-1|k_j}\|_R.$$

From the triggering condition (15), we have $\|x_{k_{j+1}-1} - \widehat{x}_{k_{j+1}-1|k_j}\|_R \leq \frac{\varepsilon_\Xi - \varepsilon_f}{\mathrm{e}^{NL_f T}} - \frac{T\varrho(T)+\xi T}{\mathrm{e}^{L_f T}}$. Then, we further obtain

$$\|x_{k_{j+1}} - \widehat{x}_{k_{j+1}|k_j}\|_R \leq \frac{\varepsilon_\Xi - \varepsilon_f}{\mathrm{e}^{(N-1)L_f T}}. \tag{19}$$

In the sequel, for clarity of exposition, the proof is divided into four aspects.

- $\overline{x}_{k_{j+1}+i|k_{j+1}} \in \mathcal{X}_i$ for all $i = 0, 1, \cdots, N-(k_{j+1}-k_j)$.

$$\begin{aligned} &\|\overline{x}_{k_{j+1}+i|k_{j+1}} - \widehat{x}_{k_{j+1}+i|k_j}\|_R \\ =&\|F_T^a(\overline{x}_{k_{j+1}+i-1|k_{j+1}}, \overline{u}_{k_{j+1}+i-1|k_{j+1}}) - F_T^a(\widehat{x}_{k_{j+1}+i-1|k_j}, \widehat{u}_{k_{j+1}+i-1|k_j})\|_R \\ \leq&\mathrm{e}^{L_f T}\|\overline{x}_{k_{j+1}+i-1|k_{j+1}} - \widehat{x}_{k_{j+1}+i-1|k_j}\|_R \\ &\cdots \\ \leq&\mathrm{e}^{iL_f T}\|x_{k_{j+1}} - \widehat{x}_{k_{j+1}|k_j}\|_R \\ \leq&\mathrm{e}^{iL_f T}\frac{\mathrm{e}^{(k_{j+1}-k_j)L_f T} - 1}{\mathrm{e}^{L_f T} - 1}(T\varrho(T) + \xi T), \end{aligned} \tag{20}$$

where the last inequality holds with substituting $i = k_{j+1} - k_j$ into the inequality (11). Since $\widehat{x}_{k_{j+1}+i|k_j} \in \mathcal{B}_{i+k_{j+1}-k_j}$, then we have $\overline{x}_{k_{j+1}+i|k_{j+1}} \in \mathcal{X}_i$ based on Lemma 4.1.

- $\overline{x}_{k_{j+1}+i|k_{j+1}} \in \mathcal{X}_i$ for all $i = N-(k_{j+1}-k_j)+1, \cdots, N-1$. First, we show $\overline{x}_{k_j+N|k_{j+1}} \in \Xi$. This guarantees that the auxiliary law $h(x)$ is allowed to be applied from t_{k_j+N}. Substituting $i = N-(k_{j+1}-k_j)$ into (20), $\|\overline{x}_{k_j+N|k_{j+1}} - \widehat{x}_{k_j+N|k_j}\|_R \leq \mathrm{e}^{(N-(k_{j+1}-k_j))L_f T}\|x_{k_{j+1}} - \widehat{x}_{k_{j+1}|k_j}\|_R$ can be obtained. Then, considering (19), we obtain $\|\overline{x}_{k_j+N|k_{j+1}} - \widehat{x}_{k_j+N|k_j}\|_R \leq \varepsilon_\Xi - \varepsilon_f$. Since $\widehat{x}_{k_j+N|k_j} \in \mathcal{X}_f$, we have $\|\overline{x}_{k_j+N|k_{j+1}}\|_R \leq \varepsilon_\Xi$, which implies $\overline{x}_{k_j+N|k_{j+1}} \in \Xi$. Then, applying the auxiliary control law $h(x)$ and according to the properties 1)–2) in Assumption 3.2, one can derive that $\overline{x}_{k_{j+1}+i|k_{j+1}} \in \mathcal{X}_f \subset \mathcal{X}_i$ for all $i = N-(k_{j+1}-k_j)+1, \cdots, N-1$.

- $\overline{x}_{k_{j+1}+N|k_{j+1}} \in \mathcal{X}_f$. As verified above, we directly obtain $\overline{x}_{k_{j+1}+N-1|k_{j+1}} \in \mathcal{X}_f$. Then, this argument is satisfied according to the property 2) of Assumption 3.2 immediately.

• $\overline{u}_{k_{j+1}+i|k_{j+1}} \in \mathcal{U}$. Since $\overline{x}_{k_{j+1}+i|k_{j+1}} \in \Xi$ for all $i = N-(k_{j+1}-k_j)+1, \cdots, N-1$ as verified above, then $h(\overline{x}_{k_{j+1}+i|k_{j+1}}) \in \mathcal{U}$ by virtue of the property 1) in Assumption 3.2. Incorporating $h(\overline{x}_{k_{j+1}+i|k_{j+1}}) \in \mathcal{U}$ and $\widehat{u}_{k_{j+1}+i|k_j} \in \mathcal{U}$, this claim completed.

These complete the proof. ∎

4.2 Stability Analysis

The stability property of the overall system under our AETMPC strategy is analyzed by the theorem given below. Hereafter, $\mathcal{X}^{MPC}$ denotes the state sets for which the solution of the OCP (13) exists.

Theorem 4.3 *Suppose that Assumption* 2.1, 3.1*, and* 3.2 *hold and the sampling period is* T*. Then, for all initial state* $x_0 \in \mathcal{X}^{MPC}$*, the overall system under the AETMPC strategy is ISpS.*

Proof We divide this proof into two parts: 1) the exact DTM of the system (1) is ISpS; 2) the solution of the overall sampled-data system is uniformly bounded over T. Then according to [20, Theorem 5], 1)+2)⇒ the overall system is ISpS.

1) To establish 1), we will show that the cost function $J_T(x(t_k), \boldsymbol{u}(t_k), N)$ is an ISpS Lyapunov function, then the same stability property of the exact DTM of the system (1) is ensured as in [21]. Let the cost function at time t_k be denoted as $J_T(x_k) = J_T(x(t_k), \boldsymbol{u}(t_k), N)$. Referring to [21], $J_T(x(t_k))$ is called an ISpS Lyapunov function, if the following

$$\alpha_1(\|x_k\|) \leq J_T(x_k) \leq \alpha_2(\|x_k\|), \tag{21}$$

$$J_T(x_{k+1}) - J_T(x_k) \leq -\alpha_3(\|x_k\|) + \gamma(\xi) + \delta \tag{22}$$

hold with a constant $\delta > 0$ and functions $\alpha_1, \alpha_2, \alpha_3 \in \mathcal{K}_\infty$, $\gamma \in \mathcal{K}$ for all $x(t_k) \in \mathcal{X}^{MPC}$, $w_{T,k} \in \mathbb{W} = \{w_T : \|w_T\|_R \leq \xi T\}$.

First of all, the lower bound of $J_T(x_k)$ is established. Considering the property 7) in Assumption 3.2, we obtain for $x_k \in \mathcal{X}^{MPC}$,

$$J_T(x_k) \geq Tl(x_k, u_k) \geq T\widetilde{\alpha}_3(\|x_k\|) =: \alpha_1(\|x_k\|).$$

Then, by virtue of the properties 3) and 5) in Assumption 3.2, it follows that

$$g(\widehat{x}_{k+N|k}) - g(\widehat{x}_{k|k}) \leq -\sum_{i=0}^{N-1} Tl(\widehat{x}_{k+i|k}, \widehat{u}_{k+i|k}) \tag{23}$$

substituting (23) into (14), we have $J_T(x_k) \leq g(\widehat{x}_{k|k}) \leq \widetilde{\alpha}_2(\|x_k\|)$ for all $x_k \in \Xi$. To get the upper bound of $J_T(x_k)$ in $\mathcal{X}^{MPC}$, we follow the idea in [8, Theorem 1]. Since $\mathcal{X}$,$\mathcal{U}$ and $\mathcal{W}$ are all compact sets, there exists a $\widetilde{J}$ such that $J_T(x_k) \leq \widetilde{J}$ for all $x_k \in \mathcal{X}^{MPC}$. Define a set $B_r \in \mathbb{R}^n, B_r = \{x \in \mathbb{R}^n : \|x\| \leq r\} \subset \mathcal{X}_f$. Let $\varepsilon = \max\left(1, \frac{\widetilde{J}}{\widetilde{\alpha}_2(r)}\right)$, we have

$$J_T(x_k) \leq \alpha_2(\|x_k\|), \quad \forall x_k \in \mathcal{X}^{MPC}$$

with $\alpha_2(\|x\|) = \varepsilon \cdot \widetilde{\alpha}_2(\|x\|)$.

Finally, we derive the difference of the cost function $J_T(x(t_k),\boldsymbol{u}(t_k),N)$ over two successive time t_{k+1} and t_k. Assume that a feasible solution $\overline{\boldsymbol{u}}(t_{k+1})$ of the OCP is constructed in the same way as in (18) based on the optimal solution $\widehat{\boldsymbol{u}}(t_k)$. Note that if the triggered condition (15) is violated at time t_k, then the OCP is solved, $J_T^0(x_k)=J_T(x(t_k),\widehat{\boldsymbol{u}}(t_k),N)$; otherwise, $J_T(x_k)=J_T(x(t_k),\overline{\boldsymbol{u}}(t_k),N)$. Thus, four scenarios should be considered, including (a) $J_T(x_{k+1})-J_T^0(x_k)$; (b) $J_T^0(x_{k+1})-J_T(x_k)$; (c) $J_T^0(x_{k+1})-J_T^0(x_k)$; and (d) $J_T(x_{k+1})-J_T(x_k)$. Since $J_T^0(x_k)\le J_T(x_k)$ and $J_T^0(x_{k+1})\le J_T(x_{k+1})$, the difference of scenario (a) is the largest of these four scenarios, we only need to focus on scenario (a). We then have

$$\begin{aligned}J_T(x_{k+1})-J_T^0(x_k)=&-Tl(\widehat{x}_{k|k},\widehat{u}_{k|k})+\sum_{l=k+1}^{k+N-1}Tl(\overline{x}_{l|k+1},\overline{u}_{l|k+1})-Tl(\widehat{x}_{l|k},\widehat{u}_{l|k})\\&+Tl(\overline{x}_{k+N|k+1},\overline{u}_{k+N|k+1})+g(\overline{x}_{k+1+N|k+1})-g(\widehat{x}_{k+N|k}).\end{aligned}\tag{24}$$

Considering the properties 3)–7), we obtain

$$\begin{aligned}&g(\overline{x}_{k+1+N|k+1})-g(\overline{x}_{k+N|k})\le -Tl(\overline{x}_{k+N|k},\overline{u}_{k+N|k}),\\&l(\overline{x}_{l|k+1},\overline{u}_{l|k+1})-l(\widehat{x}_{l|k},\widehat{u}_{l|k})\le L_l\mathrm{e}^{(l-k-1)L_fT}(T\varrho(T)+\xi T),\\&g(\overline{x}_{k+N|k})-g(\widehat{x}_{k+N|k})\le L_g\mathrm{e}^{(N-1)L_fT}(T\varrho(T)+\xi T).\end{aligned}\tag{25}$$

Substituting (25) to (24), we obtain the difference of cost function as

$$\begin{aligned}J_T(x_{k+1})-J_T(x_k)&\le J_T(x_{k+1})-J_T^0(x_k)\\&\le -\alpha_3(\|x_k\|)+\gamma(\xi)+\delta,\end{aligned}$$

where $\alpha_3(s)=T\widetilde{\alpha}_3(s)$,

$$\gamma(s)=\left[T^2L_l\mathrm{e}^{(N-1)L_fT}/(\mathrm{e}^{L_fT}-1)+TL_g\mathrm{e}^{(N-1)L_fT}\right]s$$

and

$$\delta=TL_l\mathrm{e}^{(N-1)L_fT}/(\mathrm{e}^{L_fT}-1)T\varrho(T)+L_g\mathrm{e}^{(N-1)L_fT}T\varrho(T).$$

2) Considering the property 8) in Assumption 3.2 and using the same proof technique as in [19, Remark 4.13], the uniformly bounded solution of the overall sampled-data system can be easily obtained, therefore omitted here.

Incorporating 1) and 2), the overall system is ISpS. ∎

5 Simulation Example

Consider the continuous-time cart-damper-spring system as follows

$$\begin{cases}\dot{x}_1(t)=x_2(t),\\ \dot{x}_2(t)=-\dfrac{k_0}{M}\mathrm{e}^{-x_1(t)}x_1(t)-\dfrac{h}{M}x_2(t)+\dfrac{u(t)}{M}+w(t),\end{cases}\tag{26}$$

where $x_1(t)$ is the displacement and $x_2(t)$ is the velocity. The related parameters of the system are given as: $M=1$ kg; $k_0=0.15$ N/m; $h=0.8$ N·s/m. The constraints on the state and control input are given as $\mathcal{X} = \{x : -1 \leq x \leq 1\}$ and $\mathcal{U} = \{u : -0.5 \leq u \leq 0.5\}$. The external disturbance is limited by $\|w(t)\| \leq 0.005$. For simplify, we here formulate the specific form of the disturbance-free Euler approximation of the system (26).

$$\begin{cases} x_{1,k+1} = x_{1,k} + Tx_{2,k}, \\ x_{2,k+1} = \left(1 - \dfrac{Th}{M}\right) x_{2,k} - \dfrac{k_0 T}{M} \mathrm{e}^{-x_{1,k}} x_{1,k} + \dfrac{Tu_k}{M}. \end{cases} \tag{27}$$

And the forms of other general approximations, such as modified Euler approximation and fourth-order Runge-Kutta approximation, can be seen in [22].

Considering the implementation of AETMPC strategy, the prediction horizon is set to $N=5$. Recalling the condition (16), the sampling period is selected as $T = 0.1$ s to guarantee the validity of the triggering condition. The two weighted matrices of the cost function are assumed to be $Q = [1\ 0; 0\ 1]$ and $P = 1$. By following the idea in [18], the weighted matrix R, the terminal set $\mathcal{X}_f$ and another set $\varXi$ are chosen as $R = [\begin{smallmatrix} 3.8790 & 2.1713 \\ 2.1713 & 3.222 \end{smallmatrix}]$, $\mathcal{X}_f = \{x : \|x\|_R \leq 0.7071\}$ and $\varXi = \{x : \|x\|_R \leq 0.6782\}$, respectively. The initial condition is $x_0 = [0.5, -0.8]$.

The simulation is conducted by employing Matlab subroutine fmincon. To show the validity of our AETMPC strategy, we consider three general approximations, that is, Euler approximation, modified Euler approximation and fourth-order Runge-Kutta approximation. Additionally, the robust approximation-based MPC (RAMPC) scheme in [23] is also shown here to compare with the proposed AETMPC strategy. The results are illustrated in Figures 2–4. It can be seen that the continuous-time state and control constraints are all satisfied and the overall system is ISpS under two schemes from Figures 2–3. Further, the state and control trajectories are similar under two schemes, indicating that the AETMPC strategy is comparable to those under the RAMPC scheme in terms of the constraints satisfaction. Compared with

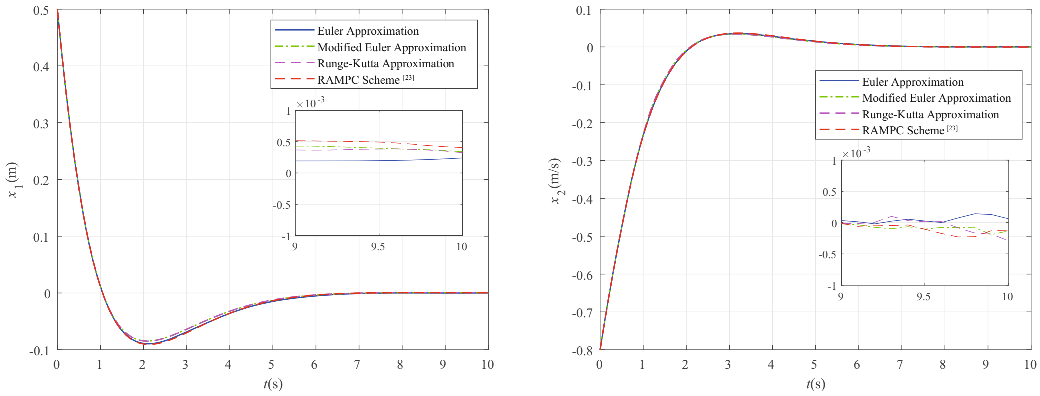

Figure 2 Comparison of system states under the AETMPC strategy (with Euler approximation, modified Euler approximation and Runge-Kutta approximation) and the RAMPC scheme

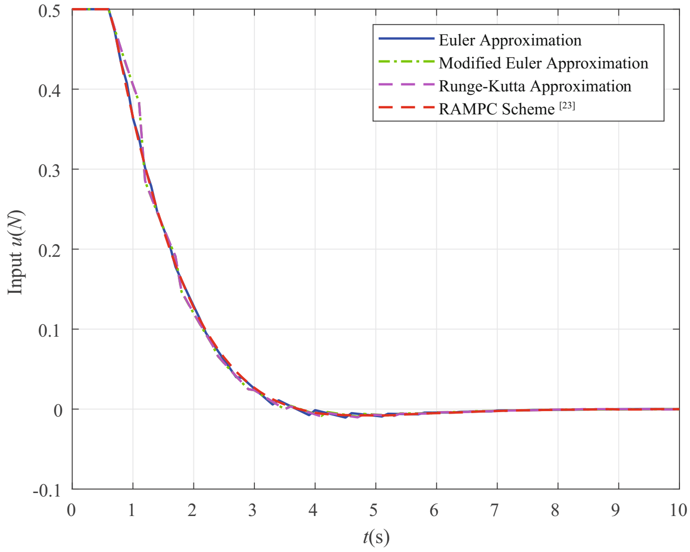

Figure 3 Comparison of the control input

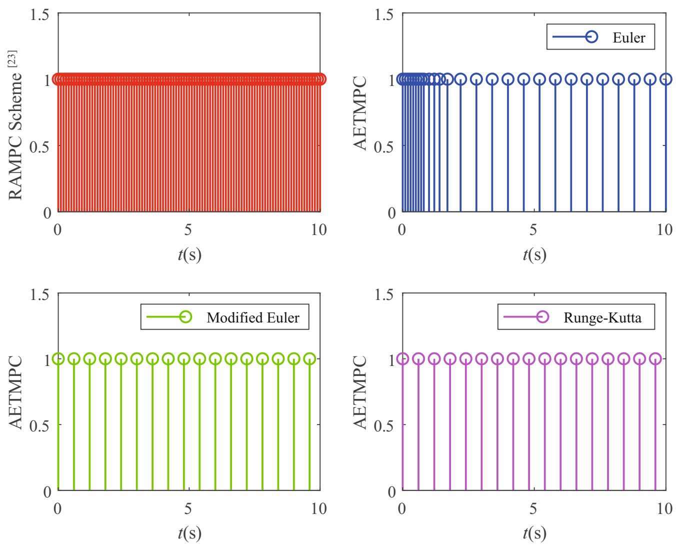

Figure 4 Comparison of triggering instants under the RAMPC scheme and the AETMPC strategy

the RAMPC scheme in [23], it can be easily observed from Figure 4 that the frequency of computing the OCP is reduced significantly by the proposed AETMPC strategy. Moreover, modified Euler approximation and fourth-order Runge-Kutta approximation are more accurate than Euler approximation, which means that the model error ($T\varrho(T)$) of the former two approximations is smaller, and hence the triggering threshold in (15) is higher, resulting in a larger triggering interval.

6 Conclusion

The continuous-time constrained nonlinear systems with disturbances have been investigated. To implement of ETMPC for continuous-time systems under the digital platform, an AETMPC strategy has been proposed, and based on which continuous-time state and control input constraints satisfaction have been achieved. More importantly, the feasibility of the AETMPC strategy has been analyzed and the ISpS property of the overall system under this strategy has been established. At last, we have verified the effectiveness of the proposed results through a numerical simulation.

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