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Robust Optimization based Air Combat Game Decision-making of Multi-UAV with Uncertain Information

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***Abstract*— The objective of this study is to design a game decision-making method for the air combat of multiple unmanned air vehicles (multi-UAV) with uncertain information, which is based on flexible and robust optimization. Aiming at the problem that the Nash equilibrium of the UAVs air combat game under uncertain information is not easy to solve, this paper proposes a new method for solving the Nash equilibrium of the air combat game under uncertain information. First of all, the interval payoff matrices of both sides are obtained according to the air combat superiority evaluation method. Secondly, the theory of obtaining the Nash equilibrium of a game by solving a linear programming problem is reviewed. Based on this idea, a novel method is proposed for solving the air combat game problem of multi-UAV with uncertain information, which is achieved by transforming it into a linear programming problem with uncertain parameters. Finally, the effectiveness of the method is verified by simulation, which shows that our method can provide new tools for solving problems in air combat with Uncertain Information.**

Keywords: Robust Optimization. Uncertain Information. Multi-UAV Air Combat. Game theory.

I. INTRODUCTION

In the complex air combat environment, due to sensor accuracy limitations, complex environment interference, packet loss in data transmission, etc, it is often difficult to directly obtain the precise parameters of targets, the adopted firepower distribution strategy, maneuver strategy, and tactical intention. Therefore, the obtained air combat information has the characteristics of uncertainty and incompleteness [1-3]. How to effectively use the incomplete air combat information obtained by UAVs and ground multi-source sensors to predict target parameters, and make autonomous decisions in UAVs air combat based on the prediction results, is of great significance for our country's UAVs in complex and changeable environments [4-7]. The key to successfully targeting enemy objectives and eventually winning the entire air combat is to use the correct tactics. At present, the research of UAVs air combat based on partial data is limited, and the existing research results are mainly divided into two types. The first type is to acquire knowledge and expand models by using data mining and data fusion, and then complete or fill the data. According to the reference [8], the air combat decision is more realistic by constructing a data mining function based on the battlefield environment. In the complicated air warfare situation, reference [9] uses the measurement error as an influencing factor to solve the fitness function value of an uncertain target and uses the IVIFS method for data fusion processing. The biggest problem of this type of method is that data mining and data fusion methods have certain limitations, which cannot guarantee the accuracy of the complete data. When many targets exist at the same time, the data mining speed is slow which cannot satisfy the real-time requirements of air combat.

The second type is to obtain the approximate value range of the target parameters, and use the optimization algorithm to transform the decision-making problem of the air combat game under the interval information into the decision-making problem of the air combat game under the certain information, so as to solve the problem [10]. Reference [11] establishes a mathematical model of interval optimization based on interval numbers, and introduces the concept of interval possibility. Reference [12] improves the TOPSIS algorithm based on the genetic algorithm, which makes the calculation result of the weight feature of the interval number more accurate.

The second type is mainly to transform the interval information obtained by the receiving equipment such as sensors into definite information through optimization model processing, and finally transform the air combat problem of incomplete information into the air combat problem under definite information. At present, there are many mathematical methods of uncertainty optimization based on interval numbers, but few of them are applied to air combat. In addition, in the complex air combat environment, the obtained interval information cannot know its specific distribution function, the interval needs to be obtained by other methods such as the interval probability. Accurate parameters cannot judge whether they meet the real situation, so it may lead to decisions which are too aggressive or conservative.

To sum up, due to the complexity of the battlefield environment, air combat situation assessment, task allocation, and maneuver decision-making under incomplete information have become an important research direction for UAVs air combat. There are many problems to be solved.

II. UNCERTAIN INFORMATION AIR COMBAT GAME PAYOFF MATRIX BASED ON INTERVAL NUMBER.

The game payoff matrix refers to the matrix formed from the game idea based on the gains or losses of both parties, which defines the selection strategy and payoff value of the game between the two parties. The benefits or utilities of different participants are the payoffs. The payoff matrix $\tilde{A}$ of the multi-UAV uncertain information air combat game based on the interval number is defined as follows:

$$\tilde{A} = \begin{array}{c} \\ \alpha_1 \\ \\ \vdots \\ \alpha_m \end{array} \begin{array}{c} \begin{matrix} \beta_1 & \beta_2 & \cdots & \beta_n \end{matrix} \\ \begin{bmatrix} [a_{11}^L, a_{11}^U] & [a_{12}^L, a_{12}^U] & \cdots & [a_{1n}^L, a_{1n}^U] \\ [a_{21}^L, a_{21}^U] & [a_{22}^L, a_{22}^U] & \cdots & [a_{2n}^L, a_{2n}^U] \\ \vdots & \vdots & & \vdots \\ [a_{m1}^L, a_{m1}^U] & [a_{m2}^L, a_{m1}^U] & \cdots & [a_{mn}^L, a_{mn}^U] \end{bmatrix} \end{array} \quad (1)$$

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where $\tilde{}$ $\quad ^{U}_{ij}$] is the payoff range of our UAVs when our UAVs adopts α_i strategy and the enemy UAVs adopts β_j strategy, m and n are the number of our UAVs strategies and the number of enemy UAVs strategies, respectively.

III. THE RELATIONSHIP BETWEEN MATRIX GAMES AND LINEAR PROGRAMMING

A strategy combination (x^*, y^*) is called a mixed-strategy Nash equilibrium, if and only if there is a number v such that $x^* = (x_1^*, x_2^*, \cdots$ is a solution of inequality (2):

$$\begin{cases} \sum_{i=1}^{m} \tilde{} \quad i = 1, 2, \cdots \\ \sum_{i=1}^{m} x_i = 1, \\ x_i \geq 0,\ i = 1, 2, \cdots \end{cases} \tag{2}$$

and $y^* = (y_1^*, y_2^*, \cdots$ is the solution of equation (3):

$$\begin{cases} \sum_{i=1}^{m} \tilde{} \quad i = 1, 2, \cdots \\ \sum_{i=1}^{m} x_i = 1, \\ x_i \geq 0,\ i = 1, 2, \cdots \end{cases} \tag{3}$$

IV. UNCERTAIN INFORMATION AIR COMBAT GAME BASED ON FLEXIBLE ROBUST OPTIMIZATION.

A. Uncertain Information Linear Programming Based on Robust Optimization.

The basic problem of linear programming under robust optimization can be described as:

$$\begin{aligned} & \max z = c^T x \\ & s.t.\ \tilde{A}x \quad b \quad \tilde{A}\ b \quad U \\ & x \in \mathrm{X} \end{aligned} \tag{4}$$

where x and c represent the vectors in the range of real numbers, X is defined as the feasible space of constraint variable x, $\tilde{A}$ is the uncertainty coefficient matrix of variable x, $\tilde{b}$ is the vector of uncertain parameters on the right-hand side of the inequality, and U is the space set of all uncertain parameters.

For any constraint in the basic linear programming problem (4), considering the i -th constraint containing uncertain parameters, the uncertain parameters $\tilde{}$ and $\tilde{}$ can be expressed as:

$$\begin{aligned} & \tilde{} \quad \xi_{ij}\hat{a}_{ij},\ \forall j \in \mathrm{J}_i \\ & \tilde{} \quad \hat{}_{0} \end{aligned} \tag{5}$$

where a_{ij} and b_i are the nominal value of uncertain parameters, $\hat{a}_{ij}$ and $\hat{b}_i$ are the positive perturbation amplitudes, ξ_{ij} and ξ_{i0} are the control variables whose size are in [-1,1] to control the fluctuation range of parameters; J_i represents the i -th constraint collection of uncertainty parameters. From equation (5), the i -th constraint in equation (4) is as follows:

$$\sum_{j \in J_i} a_{ij} x_j + \left[\max_{\xi \in U} \left\{ \sum_{j \in J_i} \xi_{ij} \hat{a}_{ij} x_j - \xi_{i0} \hat{b}_i \right\} \right] \leq b_i \tag{6}$$

According to the position of different uncertain parameters in the constraints, robust optimization problems can be divided into four categories, namely: only the left end of the constraint contains uncertain parameters, only the right end of the constraint contains uncertain parameters, both ends contain uncertain parameters, and the objective function including uncertain parameters, because the purpose of this paper is to solve the Nash equilibrium problem based on linear programming. It can be seen that after the Nash equilibrium problem is transformed into a linear programming, the constraint right-hand coefficient $\tilde{}$ is a definite value, and there are no uncertain parameters in the objective function. Therefore, the robust optimization problem belongs to the problem with only the left-end uncertainty. So the *i*-th constraint in the above linear programming problem is equivalent to:

$$\sum_{j \in J_i} a_{ij} x_j + [\max_{\xi \in U} \{ \sum_{j \in J_i} \xi_{ij} \hat{a}_{ij} x_j \}] \leq b_i \tag{7}$$

Different uncertainty sets have obvious influence on the results in the robust optimization problem. When the complexity of the uncertainty set model increases, the answer becomes increasingly complicated. When the uncertainty set's limit is wider, the ideal solution obtained is more conservative and unrealistic. The ellipsoid, box, and polyhedron are the most common uncertain sets. Polyhedral uncertain sets are more suitable for solving multi-constrained linear programming problems than the other two uncertain sets. Polyhedral uncertain sets have the following mathematical expressions:

$$U_1 = \left\{ \xi \middle| \|\xi\|_1 \leq \Gamma \right\} = \left\{ \xi \middle| \sum_{j \in J_i} |\xi_j| \leq \Gamma \right\} \tag{8}$$

where Γ is the adjustable parameter.

The robust equivalence problem of inequality (7) under polyhedral-type uncertain sets is:

$$\begin{cases} \sum_{j \in J_i} a_{ij} x_j + \Gamma p_i \leq b_i \\ p_i \geq \hat{a}_{ij} u_j, \forall j \in J_i \\ -u_j \leq x_j \leq u_j, \forall j \in J_i \end{cases} \tag{9}$$

The value range of the adjustable parameter Γ is $0 \leq \Gamma \leq j_{\max}$; $j_{\max}$ which represents the number of uncertain parameters in the i constraint. p_i and u_j are positive intermediate variables introduced for the convenience of calculation.

The set robust optimization model is to solve the problem of the maximum value of the linear programming solution, which is in line with the conventional steps to obtain the Nash equilibrium solution of the enemy. Therefore, a model for obtaining the Nash equilibrium solution of the enemy is firstly established, and the specific expression is as follows:

$$\begin{cases} \sum_{j=1}^{n} \tilde{a}_{ij} y_j \quad v \quad i=1,2,\cdots\ m \\ \sum_{j=1}^{n} y_j = 1, \\ y_j \geq 0,\ \ j=1,2,\cdots\ n \end{cases} \tag{10}$$

where $\tilde{a}_{ij} \quad a_{ij} \ ^{i} \ \vdash \xi_{ij}\hat{a}_{ij}^{1-\gamma_i}, \xi_{ij} \in [-c_{ij}, c_{ij}]$. It has been verified that the Nash equilibrium of the enemy UAVs solves the linear programming problem as follows:

$$\begin{cases} \max v \\ \sum_{j} a_{ij} y_j + \Gamma \bullet p_i \quad v \\ p_i \geq \hat{a}_{ij}\ y_j, \forall j \in J_i \\ \sum_{j=1}^{n} y_j = 1, \end{cases} \tag{11}$$

When creating a Nash equilibrium linear programming model for our air combat game with uncertain information, we must firstly transform the linear programming problem into a problem of finding the maximum value. To do so, we must find the enemy's game payoff matrix $\tilde{}$, where $\tilde{A} \qquad \tilde{A}^{T}$,and then the next steps are the same as finding the enemy's solution to Nash Equilibrium linear programming models.

B. Confidence Interval Models for Uncertain Parameters.

In the multi-UAV air combat scenario with uncertain information, it is necessary to set the perturbation interval and probability characteristics of uncertain parameters effectively.

Considering the confidence level $1-\gamma_i$, the uncertainty parameter $\tilde{a}_{ij}$ can be expressed as:

$$\tilde{a}_{ij} \quad a_{ij} \ ^{i} \ \vdash \xi_{ij}\hat{a}_{ij}^{1-\gamma_i} \tag{12}$$

where $a_{ij}^{1-\gamma_i} = \dfrac{F(\gamma_i) + F(1-\gamma_i)}{2}$, $\hat{a}_{ij}^{1-\gamma_i} = \dfrac{|F(1-\gamma_i/2) - F(\gamma_i/2)|}{2}$. $F(\bullet$ represents the inverse function of the probability distribution function of the normal distribution. The specific value can be obtained by normalizing the normal distribution and checking the standard normal distribution table. The interval range of the uncertain parameter is obtained according to the value of ξ_{ij}. $1-\gamma_i$ represents the confidence level of the uncertain parameter under the i constraint. $\hat{a}_{ij}^{1-\gamma_i}$ represents the perturbation amplitude for which the confidence level of the uncertain parameter is $1-\gamma_i$, and $\hat{a}_{ij}^{\max}$ represents the perturbation amplitude of completely credible interval. If extremely small probability values and extreme values are needed to remove, one can choose $\hat{a}_{ij}^{1-\gamma_i}$ when the confidence level is 99% as $\hat{a}_{ij}^{\max}$. Let $c_{ij} = \hat{a}_{ij}^{1-\gamma_i} / \hat{a}_{ij}^{\max}$ be the independently adjustable parameter of the uncertain parameter, and c_{ij} be the ratio between the perturbation ranges $\hat{a}_{ij}^{1-\gamma_i}$ and $\hat{a}_{ij}^{\max}$. It is easy to know that when $c_{ij} = 1$, all uncertain situations are considered to be within the interval. Similarly, if $1-\gamma_i = 0$, the interval number $\tilde{a}_{ij}$ degenerates to a constant at this time, and the robust optimization problem is transformed into a deterministic optimization problem at this time. We restrict ξ_{ij} with independently adjustable parameters, and c_{ij} can show the probability distribution characteristics of the perturbation interval. Then equation (12) can limit the range of the values of ξ_{ij}:

$$\xi_{ij} \in [-c_{ij}, c_{ij}] \tag{13}$$

The air combat game payoff matrix $\tilde{A}$ is an interval number matrix, and the interval elements in it need to be processed. The average value of the values at both ends of the interval number is set as the nominal value, and the interval number is regarded as a normal distribution with the nominal value at the center. The distribution of the interval number is inversely solved by the numerical values at both ends of the interval number. The process is as follows:

Assuming that an interval number $\tilde{}\ \qquad {}_{ij}^{U}]$ in $\tilde{A}$ follows a normal distribution $N(\mu_{ij}, \sigma_{ij}^2)$, the interval confidence level is $1-\gamma_i$, and the standard normal distribution is $N(0,1)$, through the normal distribution normalization rule:

$$\frac{a_{ij}^{U} - (a_{ij}^{L} + a_{ij}^{U})/2}{\sigma_{ij}} = \Phi^{-1}(1-\gamma_i) \tag{14}$$

So we obtain

$$\sigma_{ij} = \frac{a_{ij}^{U} - (a_{ij}^{L} + a_{ij}^{U})/2}{\Phi^{-1}(1-\gamma_i)} \tag{15}$$

where Φ^{-1} is the inverse function of the distribution function of the standard normal distribution, which can be known by checking the normal distribution table.

C. A Robust Optimization Model for Flexible Uncertain Sets.

Based on the mathematical description of uncertainty parameters in equation (12), c_{ij} is used as an independent adjustable parameter for all uncertain elements. The uncertain parameters are flexibly bounded by independent c_{ij}, because the polyhedral uncertain sets are more suitable for solving multi-constrained linear programming problems.

The establishment of a flexible polyhedron type robust optimization uncertainty set is expressed as follows:

$$U_1^{'}=\left\{\xi\left|\left\|\frac{\xi}{c}\right\|_1\le j_{\max}\right.\right\}=\left\{\xi\left|\sum_{j\in J_i}\left|\frac{\xi_{ij}}{c_{ij}}\right|\le j_{\max},c_{ij}=\frac{\hat{a}_{ij}^{1-\gamma_i}}{\hat{a}_{ij}^{\max}},\forall j\in J_i\right.\right\} \quad (16)$$

The flexibility of the flexible uncertainty set is higher than that of the traditional norm uncertainty set, and different confidence levels and perturbation ranges can be set for different sets. Compared with the traditional norm uncertainty set, the conservatism is reduced and the scale is smaller. The calculation results are more accurate.

Through the flexible and robust optimization of the uncertain set, the robust optimization problem under the flexible and uncertain set can be derived.

For the robust optimization problem under the uncertain set of flexible polyhedron type (7) is equivalent to:

$$\begin{cases}\sum_j a_{ij}^{1-\gamma_i}x_j+j_{\max}\bullet \\ p_i\ge c_{ij}\hat{a}_{ij}^{\max}u_j,\forall j\in J_i \\ -u_j\le x_j\le u_j,\forall j\in J_i\end{cases} \quad (17)$$

where u_j is a positive intermediate variable.

Based on the flexible and robust optimization problem of polyhedron type, the Nash equilibrium linear programming problem of air combat game under uncertain information is given. After verification, the linear programming problem of the enemy's Nash equilibrium solution is expressed as follows:

$$\begin{cases}\max v \\ s.t.\ \sum_j a_{ij}^{1-\gamma_i}y_j+j_{\max}\bullet \\ p_i\ge c_{ij}\hat{a}_{ij}^{\max}y_j,\forall j\in J_i \\ \sum_{j=1}^{n}y_j=1 \\ y_j\ge 0,\ j=1,2,\cdots\end{cases} \quad (18)$$

V. SIMULATION

Assume that the payoff matrix $\tilde{A}$ of the multi-UAV air combat game under our uncertain information is:

	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8
α_1	[-0.358,-0.177]	[-0.361,-0.202]	[-0.371,-0.212]	[-0.345,-0.169]	[-0.383,-0.216]	[-0.327,-0.163]	[-0.405,-0.224]	[-0.314,-0.155]
α_2	[-0.462,-0.264]	[-0.465,-0.289]	[-0.478,-0.299]	[-0.449,-0.256]	[-0.496,-0.303]	[-0.431,-0.250]	[-0.509,-0.311]	[-0.418,-0.242]
α_3	[-0.366,-0.175]	[-0.369,-0.200]	[-0.382,-0.210]	[-0.353,-0.167]	[-0.400,-0.214]	[-0.335,-0.161]	[-0.413,-0.222]	[-0.322,-0.153]
α_4	[-0.432,-0.215]	[-0.435,-0.240]	[-0.448,-0.250]	[-0.419,-0.207]	[-0.466,-0.254]	[-0.401,-0.201]	[-0.479,-0.262]	[-0.388,-0.193]
	[-0.454,-0.250]	[-0.457,-0.275]	[-0.470,-0.285]	[-0.441,-0.242]	[-0.488,-0.289]	[-0.423,-0.236]	[-0.501,-0.297]	[-0.476,-0.228]
α_6	[-0.416,-0.203]	[-0.419,-0.228]	[-0.432,-0.238]	[-0.403,-0.195]	[-0.450,-0.242]	[-0.385,-0.189]	[-0.463,-0.250]	[-0.372,-0.181]
α_7	[-0.396,-0.224]	[-0.399,-0.249]	[-0.412,-0.259]	[-0.383,-0.216]	[-0.430,-0.263]	[-0.365,-0.210]	[-0.443,-0.271]	[-0.352,-0.202]
α_8	[-0.328,-0.128]	[-0.331,-0.153]	[-0.344,-0.163]	[-0.315,-0.120]	[-0.362,-0.167]	[-0.297,-0.114]	[-0.375,-0.175]	[-0.284,-0.106]
α_9	[-0.520,-0.290]	[-0.523,-0.315]	[-0.536,-0.325]	[-0.507,-0.282]	[-0.554,-0.329]	[-0.489,-0.276]	[-0.567,-0.337]	[-0.476,-0.268]

Due to $\tilde{A}$ $\tilde{a}_{ij}$, assuming that the interval numbers in the payoff matrix obey a normal distribution, the confidence level is 0.997, and the distribution function of each interval number $\tilde{a}_{ij}$ can be inversely solved by equation(15) as:

$$\begin{bmatrix}\tilde{a} & N & & \tilde{a} & N & & \cdots a & N \\ & \vdots & & & \ddots & & & \vdots \\ \tilde{a} & N & & \tilde{a} & N & 39^2),\cdots & a & N\ \ ).372,0.038^2)\end{bmatrix}$$

According to the probability distribution, the maximum perturbation assignment $(\hat{a}_{ij}^{\max})_{9\times 8}$ is:

$$(\hat{a}_{ij}^{\max})_{9\times 8}=\begin{bmatrix}0.090 & 0.085 & 0.080 & 0.088 & 0.083 & 0.082 & 0.091 & 0.079 \\ 0.100 & 0.088 & 0.090 & 0.097 & 0.097 & 0.091 & 0.099 & 0.088 \\ 0.096 & 0.085 & 0.086 & 0.093 & 0.093 & 0.087 & 0.096 & 0.085 \\ 0.109 & 0.098 & 0.099 & 0.106 & 0.106 & 0.100 & 0.109 & 0.098 \\ 0.102 & 0.091 & 0.093 & 0.099 & 0.100 & 0.094 & 0.102 & 0.124 \\ 0.107 & 0.096 & 0.097 & 0.104 & 0.104 & 0.098 & 0.107 & 0.096 \\ 0.086 & 0.075 & 0.077 & 0.084 & 0.084 & 0.078 & 0.086 & 0.075 \\ 0.100 & 0.089 & 0.091 & 0.098 & 0.098 & 0.092 & 0.100 & 0.089 \\ 0.115 & 0.104 & 0.106 & 0.113 & 0.113 & 0.107 & 0.115 & 0.104\end{bmatrix}$$

After completing the preprocessing of the parameters, the following is based on the flexible robust optimization of polyhedron type to solve the Nash equilibrium linear programming problem with uncertain information. Since the robust optimization model set in this paper is to solve the problem of finding the maximum value of linear programming, it is in line with finding the enemy's Nash equilibrium. The equilibrium solution is a normal step, so we first obtain the enemy Nash equilibrium solution. The linear programming problem model can be known from equation (18):

$$\begin{cases} \max v \\ s.t.\ \sum_j a_{ij}^{1-\gamma_i} y_j + j_{\max} \bullet \\ p_i \geq c_{ij} \hat{a}_{ij}^{\max} y_j, \forall j \in J_i \\ \sum_{j=1}^{n} y_j = 1 \\ y_j \geq 0,\ j = 1, 2, \cdots \end{cases}$$

Bring in the parameters, get:

$$\begin{cases} \max\ v \\ -0.268y_1 - 0.282y_2 - 0.292y_3 - 0.257y_4 - 0.3y_5 - 0.245y_6 - 0.315y_7 - 0.235y_8 + 16p_1 \leq v \\ -0.363y_1 - 0.377y_2 - 0.389y_3 - 0.353y_4 - 0.4y_5 - 0.341y_6 - 0.410y_7 - 0.330y_8 + 16p_2 \leq v \\ -0.271y_1 - 0.285y_2 - 0.296y_3 - 0.257y_4 - 0.300y_5 - 0.245y_6 - 0.315y_7 - 0.235y_8 + 16p_3 \leq v \\ -0.324y_1 - 0.338y_2 - 0.349y_3 - 0.313y_4 - 0.360y_5 - 0.301y_6 - 0.371y_7 - 0.291y_8 + 16p_4 \leq v \\ -0.268y_1 - 0.282y_2 - 0.292y_3 - 0.257y_4 - 0.300y_5 - 0.245y_6 - 0.315y_7 - 0.235y_8 + 16p_5 \leq v \\ -0.310y_1 - 0.324y_2 - 0.335y_3 - 0.299y_4 - 0.346y_5 - 0.287y_6 - 0.367y_7 - 0.277y_8 + 16p_6 \leq v \\ -0.310y_1 - 0.324y_2 - 0.336y_3 - 0.300y_4 - 0.347y_5 - 0.288y_6 - 0.357y_7 - 0.277y_8 + 16p_7 \leq v \\ -0.228y_1 - 0.242y_2 - 0.254y_3 - 0.218y_4 - 0.265y_5 - 0.206y_6 - 0.275y_7 - 0.195y_8 + 16p_8 \leq v \\ -0.405y_1 - 0.419y_2 - 0.431y_3 - 0.395y_4 - 0.442y_5 - 0.383y_6 - 0.452y_7 - 0.372y_8 + 16p_9 \leq v \\ p_1 \geq 0.09y_1, p_1 \geq 0.085y_2, p_1 \geq 0.08y_3, p_1 \geq 0.088y_4, p_1 \geq 0.083y_5, p_1 \geq 0.082y_6, p_1 \geq 0.091y_7, p_1 \geq 0.079y_8 \\ p_2 \geq 0.100y_1,\ p_2 \geq 0.088y_2, p_2 \geq 0.090y_3, p_2 \geq 0.097y_4, p_2 \geq 0.097y_5,\ p_2 \geq 0.091y_6, p_2 \geq 0.099y_7\ ,p_2 \geq 0.088y_8 \\ p_3 \geq 0.096y_1,\ p_3 \geq 0.085y_2,\ p_3 \geq 0.086y_3,\ p_3 \geq 0.093y_4,\ p_3 \geq 0.093y_5,\ p_3 \geq 0.087y_6,\ p_3 \geq 0.096y_7,\ p_3 \geq 0.085y_8 \\ p_4 \geq 0.109y_1,\ p_4 \geq 0.098y_2,\ p_4 \geq 0.099y_3,\ p_4 \geq 0.106y_4,\ p_4 \geq 0.106y_5,\ p_4 \geq 0.100y_6,\ p_4 \geq 0.109y_7,\ p_4 \geq 0.098y_8 \\ p_5 \geq 0.102y_1,\ p_5 \geq 0.091y_2,\ p_5 \geq 0.093y_3,\ p_5 \geq 0.099y_4,\ p_5 \geq 0.100y_5,\ p_5 \geq 0.094y_6,\ p_5 \geq 0.102y_7,\ p_5 \geq 0.124y_8 \\ p_6 \geq 0.107y_1,\ p_6 \geq 0.096y_2,\ p_6 \geq 0.097y_3,\ p_6 \geq 0.104y_4,\ p_6 \geq 0.104y_5,\ p_6 \geq 0.098y_6,\ p_6 \geq 0.107y_7,\ p_6 \geq 0.096y_8 \\ p_7 \geq 0.086y_1,\ p_7 \geq 0.075y_2,\ p_7 \geq 0.077y_3,\ p_7 \geq 0.084y_4,\ p_7 \geq 0.084y_5,\ p_7 \geq 0.078y_6,\ p_7 \geq 0.086y_7,\ p_7 \geq 0.075y_8 \\ p_8 \geq 0.100y_1,\ p_8 \geq 0.089y_2,\ p_8 \geq 0.091y_3,\ p_8 \geq 0.098y_4,\ p_8 \geq 0.098y_5,\ p_8 \geq 0.092y_6,\ p_8 \geq 0.100y_7,\ p_8 \geq 0.089y_8 \\ p_9 \geq 0.115y_1, p_9 \geq 0.104y_2,\ p_9 \geq 0.106y_3,\ p_9 \geq 0.113y_4,\ p_9 \geq 0.113y_5,\ p_9 \geq 0.107y_6,\ p_9 \geq 0.115y_7,\ p_9 \geq 0.104y_8 \\ y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 + y_8 = 0 \end{cases}$$

Based on the improved artificial fish swarm algorithm to solve the above multi-constraint linear programming problem. The calculation result can be expressed as:

$$\begin{cases} \max\ \text{v=0.262} \\ y_1 = 0, y_2 = 0, y_3 = 0, y_4 = 0, y_5 = 0.495, y_6 = 0, y_7 = 0.505, y_8 = 0 \\ p_1 = 0.049, p_2 = 0.049, p_3 = 0.049, p_4 = 0.056, \\ p_5 = 0.052, p_6 = 0.055, p_7 = 0.043, p_8 = 0.051, p_9 = 0.057 \end{cases}$$

From the results of the above formula, it can be seen that the Nash equilibrium solution y^* of the enemy drone is: $y^* = (0,0,0,0,0.495,0,0.505,0)$, and the Nash equilibrium value of the game $V_G = 0.262$.

Since the flexible and robust optimization model defined above is to obtain the maximum value, in order to facilitate the calculation of the Nash equilibrium solution of our UAVs, the payoff matrix $\tilde{A}$ of the enemy air combat game needs to be calculated. It is easy to know $\tilde{A}$ $\tilde{A}^T$ according to the definition of the payoff matrix, and the specific solution steps are as follows the paper describes the process of finding the Nash equilibrium solution of the enemy air combat game is similar, and the Nash equilibrium solution of our UAVs is obtained by calculating the solution. It can be obtained comprehensively that our drone selects the first strategy with a probability of 0.392, the seventh strategy with a probability of 0.608, the enemy drone selects the fifth strategy with a probability of 0.495, and selects the fifth strategy with a probability of 0.505. 7 strategies, UAVs on both sides of the enemy will reach the Nash equilibrium state.

The simulation analysis shows the effectiveness of the research method in solving the multi-UAV cooperative air combat game problem under uncertain information. It can be seen that the method of the present invention improves the anti-interference ability of uncertain information processing, can effectively reduce the decision-making risk in air combat, and directly uses the game result as the decision-making basis to provide support for the decision-making of UAVs aerial uncertain air combat.

VI. COCLUSION

In this paper, a decision-making method based on flexible and robust optimization is designed. A new technology of air combat with uncertain information is designed, which aims to solve the difficult problem of Nash equilibrium of UAVs air combat game under uncertain information. According to the evaluation results of the multi-UAV air combat advantage based on uncertain information based on the interval number, the interval number benefit matrix of both sides is obtained, and then the method of solving the Nash equilibrium problem as a linear programming problem is given. The game problem of air-to-air combat is transformed into a linear programming problem with uncertain parameters. Finally, the effectiveness of the method is verified by simulation, which provides a solution to the problem of inaccurate information in air combat. Future work will design robust optimization methods for more complex air combat decision scenarios.

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