Unified Fuzzy Control of High-Order Nonlinear Systems With Multitype State Constraints

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***Abstract*—This article presents a unified adaptive fuzzy control approach for high-order nonlinear systems (HONSs) with multitype state constraints. Existing methods always require the upper and lower constraint boundaries are strictly positive and negative functions (or constants), respectively, which is often inconsistent with the actual constraints. In this article, "multitype state constraint" means that the upper and lower constraint boundaries include multiple types, such as both being strictly positive (or negative), sometime be positive or negative, and so on (cases ①–⑥). By designing a unified mapping function (UMF), the multitype state constraints are processed under removal the feasibility conditions (FCs). Furthermore, a technical design makes the proposed method also applicable to unconstrained HONSs without changing the control structure. By means of a fuzzy-logic system (FLS) and fixed-time stability theory (FTST), the proposed algorithm can ensure that the tracking error converges to a zero-centered neighborhood within a fixed time, and the singularity which often appears in the existing fixed-time control (FTC) methods of HONSs is effectively avoided. Simulation results demonstrate the scheme developed.**

***Index Terms*—Fixed-time control, fuzzy control, high-order nonlinear systems (HONSs), multitype state constraints.**

I. Introduction

In the past few decades, the control of state-constrained nonlinear systems has attracted a lot of attention because of the inevitable constraints in real systems. Just to name a few, early popular methods include reference governors [1], [2], model predictive control [3], and set invariance [4], which rely on extensive optimization calculations. Soon afterwards, Ngo et al. [5] presented a novel method based on the barrier Lyapunov function (BLF) to deal with state constraints. Inspired by this idea, many BLF-based methods were proposed for state-constrained nonlinear systems with different structures (like log-type BLF [6], [7], [8], tan-type BLF [9], [10], and integral BLF [11]). Essentially, the BLF-based approach is to convert state constraints into error constraints, thus, the so-called feasibility conditions (FCs) (first proposed in [12]) need to be satisfied. As stated in [12], users need to spend a lot of energy to explore parameters that meet the FCs through off-line optimization. To make matters worse, when the state is required to be in a set of small measures, the qualified parameters may not exist. To this end, by designing a nonlinear mapping function (NMF), Zhao and Song [12] first presented a novel tracking control method for state-constrained strict-feedback systems without FCs. Then, the NMF-based method was extended to uncertain robotic manipulators systems [13], stochastic systems [14], interconnected systems [15], and active suspension systems [16]. Recently, some methods for handling state constraints based on different NMFs were proposed successively. For example, two unified control method for nonlinear systems with or without state constraints were proposed in [17] and [18]. Zhao et al. [19] presented a unified control approach to strict-feedback nonlinear systems under dynamic constraints, this method creatively removes the restriction of the traditional method on constraint boundaries. Recently, the method was applied to uncertain robotic systems [20].

It is noteworthy that all the above methods focus on general feedback nonlinear systems. However, as a special class of nonfeedback linearization systems, the control design of high-order nonlinear systems (HONSs) is more trickier, especially, in the case of state constraints. For all this, some breakthroughs have been made in recent years. Among them, the method based on high-order BLF (HOBLF) has attracted much attention. For instance, by designing the tan-type HOBLFs, the methods for symmetric/asymmetric output-constrained HONSs were presented in [21], [22], and [23]. Whereafter, the approaches based on tan-type HOBLFs of full-state constrained HONSs were proposed in [24], [25], and [26]. In addition, by constructing the log-type HOBLFs, the control schemes were presented for state-constrained HONSs [27], [28], [29], [30]. Similar to the BLF-based method to deal with the state constraints of general feedback nonlinear systems, the HOBLF-based method still faces the limitation of FCs. Toward this end, the NMF-based approach was extended to state-constrained

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HONSs [31], [32], [33], which does not involve FCs. It is worth noting that one limitation of the above methods is that the sign of the constraint boundaries are definite, that is, the upper and lower constraint boundaries need to be strictly positive and strictly negative, respectively. In addition, the schemes proposed in [27], [28], [29], [30], [31], and [32] are invalid for unconstrained HONSs. In fact, for the constraint problem $L_i(t) < x_i < H_i(t)$, where $L_i(t), H_i(t) \in L_\infty$ denote the constraint boundaries, x_i denotes system variable. There are many cases of the sign of constraint boundaries, as follows:

①$L_i(t) < 0 < H_i(t)$

②$L_i(t) \leq 0 < H_i(t)$

③$L_i(t) < 0 \leq H_i(t)$

④$L_i(t) < H_i(t) \leq 0$

⑤$0 \leq L_i(t) < H_i(t)$

⑥$L_i(t) < H_i(t)$, where $L_i(t)$ and $H_i(t)$ could sometime be positive or negative.

Furthermore, the actual system may need to operate interactively under constrained and unconstrained conditions. In other words, the constraint boundaries are bounded for some specific periods of time and unbounded for the rest of the time, like $L_i(t) = -\infty, H_i(t) = +\infty$ when $0 \leq t \leq t_0$, $L_i(t), H_i(t) \in L_\infty$ when $t > t_0$. The traditional schemes of state-constrained HONSs [24], [25], [26], [27], [28], [29], [30], [31], [32], [33] are only applicable to case ①, and the methods in [27], [28], [29], [30], [31], and [32] are invalid for unconstrained HONSs. Note that the schemes proposed in [24], [25], [26], and [33] are also valid for unconstrained HONSs, but the limitation that the upper and lower constraint boundaries are positive and negative functions, respectively, still exists, and the schemes proposed in [24], [25], and [26] rely on the FCs. This impels us to develop a more general adaptive control method for HONSs subject to multitype state constraints (cases ①–⑥), which is also applicable to unconstrained HONSs.

As we all know, the settling time of the actual system is an important indicator of user concern. Nevertheless, the early methods can only ensure the stability of the system without much consideration of the settling time, which in fact was often infinite. In order to meet the user's higher requirements for settling time, a large number of finite-time control methods based on finite-time stability theory [34] were developed for output/state-constrained feedback nonlinear systems [35], [36] and HONSs [21], [37], [38]. One blemish of the approaches mentioned above is that the settling time is related to the selection of the initial value of the system. Toward this end, the fixed-time stability theory (FTST) which can remove the above defect was proposed in [39], with its help, the settling time is only related to the design parameters. Recently, many fixed-time control (FTC) schemes were presented, see [40], [41], [42], [43], [44], [45], [46], [47] and references therein. As far as we know, most of the existing methods focus on general feedback nonlinear systems except for those in [45], [46], and [47], while the schemes proposed in [45] and [46] depend on the requirement of more prior knowledge of system functions (see Remark 3), and the singularity problem may appear in the method presented in [47] (see Remark 10), which impels us to construct a nonsingular FTC strategy of state-constrained HONSs to remove the above restrictions.

According to the above analysis, one can know that a lot of progress has been developed for state-constrained HONSs [24], [25], [26], [27], [28], [29], [30], [31], [32], [33], but the following limitations still exist.

1) The tan/log-type HOBLFs-based approaches for HONSs [24], [25], [26], [27], [28], [29], [30] involves FCs. The NMF-based approaches for HONSs [31], [32], [33] remove the FCs, however, all of these methods require that the sign of the constraint boundaries be definite (i.e., case ①), and the methods in [31] and [32] are invalid for unconstrained HONSs.
2) Most of the control approaches for HONSs only ensure that the settling time of HONSs is infinite, or the settling time is related to the selection of the initial value except for those in [45], [46], and [47], while the schemes proposed in [45] and [46] depend on the requirement of more prior knowledge of system functions (see Remark 3), and the singularity problem may arise in the approach presented in [47] (see Remark 10).

The above analysis impels us to construct a unified nonsingular FTC algorithm of HONSs with multitype state constraints (cases ①–⑥) or without state constraints. Conclude the key innovations as follows.

1) A unified adaptive control algorithm of HONSs subject to multitype state constraints is presented in this article. By designing a unified mapping function (UMF), the HONS subject to multitype state constraints is rendered into an unconstrained one. The difference from the conventional approaches [24], [25], [26], [27], [28], [29], [30], [31], [32], [33] is that the proposed scheme effectively eliminates the FCs, and capable of accommodating multiple type time-varying state constraints (cases ①–⑥). Moreover, a technical design makes the proposed method also suitable for unconstrained HONSs.
2) With the aid of a fuzzy-logic system (FLS) and FTST, a nonsingular FTC approach for HONSs subject to multitype state constraints is proposed. Under the proposed strategy, all of the closed-loop signals are bounded, and the tracking error converges to a zero-centered neighborhood within a fixed time. Moreover, the limitation of existing finite/fixed-time methods [21], [37], [38], [45], [46] that rely on more prior knowledge of $f_i(\cdot)$ and $g_i(\cdot)$ is removed, and the singularity problem which appears in the existing FTC method for HONSs [33], [47] is also effectively removed.

II. Problem Formulation and Preliminaries

A. Preliminaries

Lemma 1 [41]: Give the following nonlinear system:

$$\dot{x} = f(x, t). \tag{1}$$

For a positively definite radially unbounded Lyapunov function (LF) $V(x)$, if $\exists\ \lambda_1, \lambda_2, \Delta \in R^+, r_1 \in (0, 1), r_2 \in (1, +\infty)$,

$\eta \in (0, 1)$ such that

$$\dot{V}(x) \le -\lambda_1 V^{r_1}(x) - \lambda_2 V^{r_2}(x) + \Delta \tag{2}$$

then, the solution of (1) is practical fixed-time stable, and the residual set of the solution for (1) is

$$x \in \left\{ x \middle| V \le \min\left\{ \left(\frac{\Delta}{(1-\eta)\lambda_1}\right)^{\frac{1}{r_1}}, \left(\frac{\Delta}{(1-\eta)\lambda_2}\right)^{\frac{1}{r_2}} \right\} \right\} \tag{3}$$

with

$$T \le \frac{1}{\lambda_1 \eta (1 - r_1)} + \frac{1}{\lambda_2 \eta (r_2 - 1)}. \tag{4}$$

Lemma 2 [7]: The FLS can be applied to approximate an unknown nonlinear function $\bar{f}(\mathcal{Z})$, that is

$$\bar{f}(\mathcal{Z}) = \Upsilon^T \Omega(\mathcal{Z}) + \epsilon(\mathcal{Z}), \quad \left(|\epsilon(\mathcal{Z})| \le \epsilon, \epsilon \in R^+\right) \tag{5}$$

where $\Omega(\mathcal{Z}) = [\Omega_1(\mathcal{Z}), \ldots, \Omega_m(\mathcal{Z})]^T / \sum_{i=1}^{n} \Omega_i(\mathcal{Z})$, $m \in R^+$ denotes the number of the fuzzy rules, $\mathcal{Z}$, $\Omega(\mathcal{Z})$, Υ, and $\epsilon(\mathcal{Z})$ denote input, basis function, weight, and error, respectively. Choose $\Omega_i(\mathcal{Z})$ as follows:

$$\Omega_i(\mathcal{Z}) = \exp\left[\frac{-(\mathcal{Z} - \vartheta_i)^T (\mathcal{Z} - \vartheta_i)}{\iota_i^2}\right], i = 1, \ldots, m \tag{6}$$

where ι_i and ϑ_i denote the center vector and the spreads of $\Omega_i(\mathcal{Z})$.

Remark 1: It is noted that the existing popular FLSs that by using the singleton fuzzifier, product inference, center-average defuzzifier, and Gaussian membership function, the output of an FLS is in a form of a normalized Gaussian function is different from the ones adopted in this article. The method in this article only estimates the vector norm to make the design and analysis process simpler, and the similar methods are also proposed in [7], [9], [10], and [11]. Of course, this approach will also affect the control strength and transient performance to a certain extent.

Lemma 3 [22]: For $p \in R^{\ge 1}_{\text{odd}} = \{p \ge 1 | p = q_1/q_2\}$, $q_i \in R^+ (i = 1, 2)$ stands for odd, $m, n \in R$, $x, y, \zeta, \Gamma \in R^+$, one has

$$\left|m^p - n^p\right| \le \Xi |m - n| \left(|m - n|^{p-1} + n^{p-1}\right) \tag{7}$$

$$\Gamma |m|^x |n|^y \le \frac{\zeta x}{x + y} |m|^{x+y} + \frac{y \Gamma^{\frac{x+y}{y}}}{x + y} \zeta^{-\frac{x}{y}} |n|^{x+y} \tag{8}$$

where $\Xi = p(2^{p-2} + 2)$.

Lemma 4 [33]: $\forall\ \chi, \chi_i \in R$, $\varepsilon \in R^+$, one has

$$0 \le |\chi| < \varepsilon + \frac{\chi^2}{\sqrt{\chi^2 + \varepsilon^2}} \tag{9}$$

$$\left(\sum_{i=1}^{n} |\chi_i|\right)^{\kappa} \le \max\left\{1, n^{\kappa - 1}\right\} \sum_{i=1}^{n} |\chi_i|^{\kappa}. \tag{10}$$

Lemma 5 [51]: For $x \ge 0, y > 0$ and $a \ge 1$, one has

$$x \le y + \left(\frac{x}{a}\right)^a \left(\frac{a-1}{y}\right)^{a-1}. \tag{11}$$

B. Problem Formulation

Consider the following HONSs:

$$\begin{cases} \dot{x}_i = f_i(\bar{x}_n) + g_i(\bar{x}_n) x_{i+1}^{p_i}, i = 1, \ldots, n-1 \\ \dot{x}_n = f_n(\bar{x}_n) + g_n(\bar{x}_n) u^{p_n} \\ y = x_1 \end{cases} \tag{12}$$

subject to full-state constraints as follows:

$$x_i \in \mathcal{A}_i := \left\{(t, x_i) \in R^+ \times R \middle| L_i(t) < x_i < H_i(t)\right\} \tag{13}$$

where $-\infty < L_i(t) < H_i(t) < +\infty$ with $i = 1, \ldots, n$, $L_i(t)$ and $H_i(t)$ denote the continuously differentiable time-varying constraint boundaries. $y, u, \bar{x}_i = [x_1, \ldots, x_i]^T$ denote system output, system input, and state vector, respectively. $p_i \in R^{\ge 1}_{\text{odd}}$, $f_i(\cdot), g_i(\cdot)$ $(i = 1, \ldots, n)$ denote unknown nonlinear functions.

Remark 2: It is noteworthy that the constraint boundaries considered in the existing methods are very conservative, that is, the upper and lower constraint boundaries are strictly positive and negative functions (or constants), respectively, $(-\infty < L_i(t) < 0 < H_i(t) < +\infty)$. In fact, the constraint boundaries of the actual system contain many types (cases ①–⑥). For example, if the reference signal is strictly positive (or strictly negative), like $y_d = 0.5 + 0.3\sin(t)$ [or $y_d = -0.5 + 0.3\sin(t)$], in this case, it is meaningless to require the lower constraint boundary (or upper constraint boundary) to be a negative function (or positive function), and the constraint range of system variables may be strictly positive (or strictly negative) according to actual needs. In addition, even though the reference signal is sometimes positive and sometimes negative, like $y_d = \sin(t)$, some practical systems may require that the constraint boundaries be sometimes positive and sometimes negative, like $L_1(t) = \sin(t) - 0.5$ and $H_1(t) = \sin(t) + 0.5$. Furthermore, as stated in the introduction, the actual system may need to run interactively under constrained and unconstrained conditions. To the best of our knowledge, the unified control of HONSs with multitype state constraints (cases ①–⑥) or without state constraints is also an open issue, and this article is dedicated to solving this valuable problem.

The aim of this article is to develop a unified fuzzy control strategy such that: 1) all of the closed-loop signals are bounded; 2) the tracking error $z_1 = y - y_d$ converges to a zero-centered neighborhood within a fixed time, where y_d denotes a known desired signal; and 3) the full-state constraints are not violated under removal FCs.

Two rationalization assumptions are needed.

Assumption 1: $g_n(\cdot)$ is strictly positive or strictly negative, and there exist two positive constants $\underline{g}, \bar{g}$, such that $0 < \underline{g} \le |g_n(\cdot)| \le \bar{g}$. In this article, let $g_n(\cdot) > 0$.

Assumption 2 [34]: The desired signal y_d and its (n-1)th derivative $y_d^{(n-1)}$ are bounded and continuous.

Remark 3: Assumption 1 is a standard assumption used for stability analysis. Obviously, Assumption 1 is easily satisfied in many actual systems, such as single-link robotic system, reduced-order model of the boiler-turbine unit, and so on. Note that most existing methods require that the sign of each control gain be given, and this article only requires that the sign of $g_n(\cdot)$ be given after technical processing. Furthermore, the existing finite/fixed-time control methods of HONSs [21], [37], [38], [45], [46] depend on the requirement of more prior knowledge of $f_i(\cdot)$ (see Assumption 1 of [37], [38], [45] and Assumption 2 of [21], [46]), but the above assumptions are also eliminated in this article. Assumption 2 is a standard assumption, which is also given in [12] and [41].

III. Main Results

A. Unified Mapping Function

Inspired by [19], an important UMF is designed as follows:

$$\xi_i = \frac{H_i(t)x_i - \underline{H}_i}{2(H_i(t) - x_i)} - \frac{L_i(t)x_i + \bar{L}_i}{2(x_i - L_i(t))}, i = 1, \ldots, n \quad (14)$$

with $x_i(0) \in \mathcal{A}_i$, where $\bar{L}_i$ and $\underline{H}_i$ denote some constants that satisfy the following conditions:

$$L_i^2(t) + \bar{L}_i > 0, \quad H_i^2(t) - \underline{H}_i > 0. \quad (15)$$

According to (14) and (15), it is easy to see that the UMF ξ_i has the following properties: 1) if $x_i \in \mathcal{A}_i$, then $\xi_i \to \infty$ if and only if $x_i \to L_i^+(t)$ or $x_i \to H_i^-(t)$ and 2) $\lim_{L_i(t)\to-\infty, H_i(t)\to+\infty} \xi_i = x_i$.

Then, one has the following Lemma, which is crucial for subsequent analysis.

Lemma 6: For the initial condition $x_i(0) \in \mathcal{A}_i$, if $\xi_i(t) \in L_\infty$, then $x_i(t) \in \mathcal{A}_i$ holds for $\forall t \in [0, +\infty)$.

Proof: Suppose that $x_i(t^*) \notin \mathcal{A}_i$, that is, $\exists t^* \in (0, +\infty)$ s.t. $x_i(t^*) = a$, where $a \le L_i(t)$ or $a \ge H_i(t)$. According to $x_i(0) \in \mathcal{A}_i$, and the continuity of $x_i(t)$, one can obtain from the intermediate value theorem that there is at least one time $t_1 \in (0, t^*)$ such that $x_i(t_1) = L_i(t_1)$ or $x_i(t_1) = H_i(t_1)$. Then, one has $\xi_i(t_1) = \infty$, which obviously contradicts $\xi_i \in L_\infty$. So, Lemma 6 holds. ■

Remark 4: In the existing methods [24], [25], [26], [27], [28], [29], [30], take the log-type HOBLF, for example, for simplicity, $-L_i(t) = H_i(t) = B_i$ denotes a positive constant (i.e., symmetric constant constraints), the log-type HOBLF is as follows:

$$V_i = \frac{1}{P_i}\log\left(\frac{k_i^{P_i}}{k_i^{P_i} - z_i^{P_i}}\right), \quad |z_i(0)| < k_i \quad (16)$$

where k_i denotes a positive constant, $z_i = x_i - \alpha_{i-1}$, $P_i = p - p_i + 2$, and $p = \max\{p_i\}$, k_i denotes a positive constant, which will be given below. If the following conditions are satisfied:

$$|z_i(t)| < k_i \quad \forall t \ge 0 \quad (17)$$

$$k_1 = B_1 - A_0 > 0, \quad |y_d| \le A_0 \quad (18)$$

$$k_i = B_i - A_{i-1} > 0, i = 2, \ldots, n \quad (19)$$

$$|\alpha_{i-1}| \le A_{i-1} < B_i, i = 2, \ldots, n \quad (20)$$

then the state constraints are not violated, where A_i denotes a constant. In fact, the main idea of above schemes is to transform state constraints into error constraints. It can be known from (17) and the expression of z_i that $y_d(0) - k_1 < x_1(0) < k_1 + y_d(0)$ and $\alpha_{i-1}(0) - k_i < x_i(0) < k_i + \alpha_{i-1}(0), (i = 2, \ldots, n)$, it means that the system variables x_i are required to be in some sets of small measures. Moreover, it can be known from (20) that the so called "FC" need to be satisfied, that is, $|\alpha_{i-1}| < B_i (i = 2, \ldots, n)$. It can be obviously obtained from (14) that the proposed UMF ξ_i is purely dependent on system variables, so the FC is removed.

Remark 5: Recently, several approaches to removing the FC have been proposed for state-constrained feedback nonlinear systems [12], [13], [14], [15], [16], [17], [18], [19], [20] and HONSs [31], [32], [33]. For instance, the NMF presented in [12], [13], [14], [15], [16], [31], and [32] is as follows:

$$\xi_i = \frac{x_i}{(x_i - L_i(t))(H_i(t) - x_i)}. \quad (21)$$

From (21), one has

$$\dot{\xi}_i = \mu_i \dot{x}_i + \nu_i \quad (22)$$

where

$$\mu_i = \frac{x_i^2 - L_i(t)H_i(t)}{(x_i - L_i(t))^2(H_i(t) - x_i)^2} \quad (23)$$

$$\nu_i = \frac{\big(H_i(t)\dot{L}_i(t) + \dot{H}_i(t)L_i(t)\big)x_i - \big(\dot{L}_i(t) + \dot{H}_i(t)\big)x_i^2}{(x_i - L_i(t))^2(H_i(t) - x_i)^2}. \quad (24)$$

From (23), it can be known that $\mu_i = 0$ might be true if $H_i(t)$ and $L_i(t)$ have the same sign or one of them is zero, so that system (22) becomes uncontrollable. Thus, the methods of [12], [13], [14], [15], [16], [31], and [32] are only applicable to case ① (i.e., $-\infty < L_i(t) < 0 < H_i(t) < +\infty$). Furthermore, from (21), one has $\lim_{L_i(t)\to-\infty, H_i(t)\to+\infty} \xi_i = 0$, which means that the above schemes are invalid for unconstrained HONSs. The NMF presented in [17] is as follows:

$$\xi_i = \frac{-L_i H_i x_i}{(x_i - L_i)(H_i - x_i)} \quad (25)$$

where L_i and H_i are constants.

From (25), one has

$$\dot{\xi}_i = \mu_i \dot{x}_i \quad (26)$$

where

$$\mu_i = \frac{H_i L_i\big(L_i H_i - x_i^2\big)}{(x_i - L_i)^2(H_i - x_i)^2}. \quad (27)$$

From (27), it can be known that $\mu_i = 0$ might be true if H_i and L_i have the same sign or if one of them is zero. Similar to the above analysis, we can see that the method proposed in [17] is also only applicable to case ①, and this method is limited to constant constraints. The NMF presented in [18] and [33] is as follows:

$$\xi_i = \frac{x_i}{\hbar_{i1}(x_i, L_i(t))\hbar_{i2}(x_i, H_i(t))} \quad (28)$$

where

$$\hbar_{i1}(x_i, L_i(t)) = -\mathcal{C}_i(x_i)\left(\frac{x_i}{L_i(t)}\right)^{2m} + 1 \quad (29)$$

$$\hbar_{i2}(x_i, H_i(t)) = -(1 - \mathcal{C}_i(x_i))\left(\frac{x_i}{H_i(t)}\right)^{2m} + 1$$

$$\mathcal{C}_i(x_i) = \begin{cases} 1, & L_i(t) < x_i \le 0 \\ 0, & 0 < x_i < H_i(t) \end{cases} \quad (30)$$

and $2m > n, m \in Z^+$. Obviously, the original design of the above methods is based on the fact that the upper and lower constraint boundaries are positive and negative functions, respectively, so they are also only applicable to case ①.

Furthermore, The fixed-time controller designed by the method in [33] has the singularity problem, while the proposed method effectively removes the singularity problem (see Remark 10).

Remark 6: Note that the methods of [19] and [20] can deal with constraints including cases ①–⑥ (i.e., $-\infty < L_i(t) < H_i(t) < +\infty$), but the considered systems are limited to feedback nonlinear systems. In addition, the proposed method has another advantage that is not found in [19] and [20], that is, the proposed method is suitable for unconstrained HONSs. The NMF presented in [19] and [20] is as follows:

$$\xi_i = \frac{x_i - \underline{H}_i}{H_i(t) - x_i} + \frac{x_i - \bar{L}_i}{x_i - L_i(t)}, i = 1, \ldots, n \tag{31}$$

where $\bar{L}_i$ and $\underline{H}_i$ denote some constants that satisfy the following conditions $L_i(t) < \bar{L}_i, H_i(t) > \underline{H}_i$. Obviously, $\lim_{L_i(t)\to-\infty, H_i(t)\to+\infty} \xi_i = 0$. However, it can be obtained from (14) that $\lim_{L_i(t)\to-\infty, H_i(t)\to+\infty} \xi_i = x_i$, it means that the proposed scheme works just as well when there are no constraints.

Based on (14), one has

$$\dot{\xi}_i = \mu_i \dot{x}_i + v_i, i = 1, \ldots, n \tag{32}$$

where

$$\mu_i = \frac{L_i^2(t) + \bar{L}_i}{2(x_i - L_i(t))^2} + \frac{H_i^2(t) - \underline{H}_i}{2(H_i(t) - x_i)^2} \tag{33}$$

$$v_i = -\frac{\dot{L}_i(t)x_i^2 + \dot{L}_i(t)\bar{L}_i}{2(x_i - L_i(t))^2} + \frac{-\dot{H}_i(t)x_i^2 + \dot{H}_i(t)\underline{L}_i}{2(H_i(t) - x_i)^2}. \tag{34}$$

Substituting (12) to (32), one can obtain

$$\begin{cases} \dot{\xi}_i = F_i(\bar{x}_n, \xi_{i+1}) + \xi_{i+1}^{p_i}, i = 1, \ldots, n-1 \\ \dot{\xi}_n = F_n(\bar{x}_n) + \mu_n g_n(\bar{x}_n) u^{p_n} \end{cases} \tag{35}$$

where $F_i(\bar{x}_n, \xi_{i+1}) = \mu_i[g_i(\bar{x}_n)x_{i+1}^{p_i} + f_i(\bar{x}_n)] - \xi_{i+1}^{p_i} + v_i$ and $F_n(\bar{\xi}_n) = \mu_n f_n(\bar{x}_n) + v_n$.

Remark 7: Obviously, it can be known from (33) that as long as (15) is true, then $\mu_i \neq 0$ is true, and it does not matter what the sign of the constraint boundaries are, which indicates that the proposed method can cover all of six-type constraint cases. According to Lemma 6, as long as the designed controller can ensure that the variable ξ_i in system (35) is bounded, then the conclusion that the state constraints are not violated is true.

Remark 8: It can be known from (14) that ξ_i is a strictly monotone function of x_i when $x_i \in (L_i(t), H_i(t))$, which means that the inverse function exists, that is, every ξ_i is actually unique corresponding to a x_i, but the concrete expression can not be expressed concretely. Indeed, this has no effect on our results. In the process of designing the controller, we will use the FLS to approximate the unknown nonlinear function, so only the variables included in the nonlinear function as input, there is no requirement for the specific function expression.

B. Controller Design

Define the error variable as follows:

$$e_i = \xi_i - \alpha_{i-1}, i = 1, \ldots, n \tag{36}$$

where

$$\alpha_0 = \frac{H_1(t)y_d - \underline{H}_1}{2(H_1(t) - y_d)} - \frac{L_1(t)y_d + \bar{L}_1}{2(y_d - L_1(t))}$$

and $\alpha_{i-1} (i = 2, \ldots, n)$ will be given below.

According to (35) and (36), we have

$$\begin{cases} \dot{e}_i = F_i(\bar{x}_n, \xi_{i+1}) + \xi_{i+1}^{p_i} - \dot{\alpha}_{i-1}, i = 1, \ldots, n-1 \\ \dot{e}_n = F_n(\bar{x}_n) + \mu_n g_n(\bar{x}_n) u^{p_n} - \dot{\alpha}_{n-1}. \end{cases} \tag{37}$$

Let $\Theta_i = ||\Upsilon_i||^2$, Υ_i stands for the weight vector of FLS, $\tilde{\Theta}_i = \Theta_i - \hat{\Theta}_i$, $\hat{\Theta}_i$ denotes the estimate of Θ_i, $Z_i = [\bar{x}_n, \bar{\xi}_n, y_d, \dot{y}_d, \ldots, y_d^{(i-1)}, \hat{\Theta}_i]^T$, $\mathcal{Z}_i = [\bar{x}_i, \bar{\xi}_i, y_d, \ldots, y_d^{(i-1)}, \hat{\Theta}_i]^T$, $p = \max_{1\le i\le n}\{p_i\}$, and $P_i = p - p_i + 2$. According to the backstepping technique, the specific design process of the controller is as follows.

Step 1: Choose the first LF candidate (LFc) V_1 as follows:

$$V_1 = \frac{1}{P_1} e_1^{P_1} + \frac{1}{2\gamma_1} \tilde{\Theta}_1^2, \quad (\gamma_1 \in R^+). \tag{38}$$

From (37) and (38), we have

$$\begin{aligned} \dot{V}_1 = {} & e_1^{P_1-1} \bar{f}_1(Z_1) + e_1^{P_1-1}(\xi_2^{p_1} - \alpha_1^{p_1}) + e_1^{P_1-1}\alpha_1^{p_1} \\ & - \frac{P_1 - 1}{P_1} e_1^{P_1} - (\psi_1 + \lambda_{12}\varepsilon_1) e_1^{p+1} - \frac{1}{\gamma_1}\tilde{\Theta}_1 \dot{\hat{\Theta}}_1 \end{aligned} \tag{39}$$

where $\bar{f}_1(Z_1) = F_1(\bar{x}_n, \xi_2) + (P_1 - 1)e_1/P_1 - \dot{\alpha}_0 + (\psi_1 + \lambda_{12}\varepsilon_1)e_1^{p_1}$, $\lambda_{12}, \varepsilon_1 \in R^+$, ψ_1 will be given below.

Based on Lemma 2, an FLS is applied to approximate $\bar{f}_i(Z_i)$, that is, $\bar{f}_i(Z_i) = \Upsilon_i^T \Omega_i(Z_i) + \epsilon_i(Z_i)$, where $|\epsilon_i(Z_i)| \le \epsilon_i$ with $\epsilon_i \in R^+$.

Combine Lemmas 3 and 5 with the conclusion that $||\Omega_i(\cdot)||^2 \in (0, 1]$, one has

$$\begin{aligned} e_1^{P_1-1}\bar{f}_1(Z_1) \le {} & \frac{|e_1|^{P_1-1}\tilde{\Theta}_1}{4a_1||\Omega_1(\mathcal{Z}_1)||^2} + |e_1|^{P_1-1}\Psi_1 + e_1^{P_1-1}\epsilon_1(Z_1) \\ \le {} & \frac{|e_1|^{P_1-1}\tilde{\Theta}_1}{4a_1||\Omega_1(\mathcal{Z}_1)||^2} + \delta_1 + e_1^{p+1}\phi_1 \\ & + \frac{P_1 - 1}{P_1} e_1^{P_1} + \frac{\epsilon_1^{P_1}}{P_1} \end{aligned} \tag{40}$$

where $\Psi_1 = (\sqrt{1 + \hat{\Theta}_1^2})/(4a_1||\Omega_1(\mathcal{Z}_1)||^2) + a_1$, $\phi_1 = [(P_1 - 1)\Psi_1/(p+1)]^{(p+1)/(P_1-1)}[p_1/(P_1 - 1)\delta_1]^{p_1/(P_1-1)}$. a_1 and δ_1 are positive design parameters.

From Lemma 3, one has

$$e_1^{P_1-1}(\xi_2^{p_1} - \alpha_1^{p_1}) \le \frac{p_1 + 1}{p + 1} e_2^{p+1} + \psi_1 e_1^{p+1} \tag{41}$$

where $\Xi_1 = p_1(2^{p_1-2} + 2)$ and $\psi_1 = [(P_1 - 1)\Xi_1^{(p+1)/(P_1-1)}]/(p+1) + [(p+2)(\Xi_1|\pi_1|^{p_1-1})^{(p+1)/(p+2)}]/(p+1)$.

Based on (40) and (41), one can obtain

$$\begin{aligned} \dot{V}_1 \le {} & e_1^{P_1-1}\alpha_1^{p_1} + \frac{|e_1|^{P_1-1}\tilde{\Theta}_1}{4a_1||\Omega_1(\mathcal{Z}_1)||^2} + \phi_1 e_1^{p+1} - \lambda_{12}\varepsilon_1 e_1^{p+1} \\ & + \frac{p_1 + 1}{p + 1} e_2^{p+1} + \Delta_1 - \frac{1}{\gamma_1}\tilde{\Theta}_1\dot{\hat{\Theta}}_1 \end{aligned} \tag{42}$$

where $\Delta_1 = \delta_1 + \epsilon_1^{P_1}/P_1$.

Design α_1 and $\dot{\hat{\Theta}}_1$ as follows:

$$\alpha_1 \triangleq -\Lambda_1 e_1 \tag{43}$$

$$\Lambda_1 = \left[\lambda_{11} + \frac{\lambda_{12} e_1^{2(P_1 r_2 - p - 1)}}{\sqrt{e_1^{2(P_1 r_2 - p - 1)} + \varepsilon_1^2}} + \phi_1\right]^{\frac{1}{p_1}} \tag{44}$$

$$\dot{\hat{\Theta}}_1 = \frac{\gamma_1 |e_1|^{P_1 - 1}}{4a_1 ||\Omega_1(\mathcal{Z}_1)||^2} - \sigma_1 \hat{\Theta}_1, \left(\hat{\Theta}_1(0) > 0\right) \quad (45)$$

where $r_2 > \max\{1, (3/2 + p)/P_i\}$, $\lambda_{11}, \sigma_1 \in R^+$.

Based on Lemma 4, one has

$$e_1^{P_1 - 1}\alpha_1^{p_1} \leq -\lambda_{11} e_1^{p+1} - \phi_1 e_1^{p+1} + \lambda_{12}\varepsilon_1 e_1^{p+1} - \lambda_{12} e_1^{P_1 r_2} \quad (46)$$

Substituting (45) and (46) to (42), one has

$$\dot{V}_1 \leq -\lambda_{11} e_1^{p+1} - \lambda_{12} e_1^{P_1 r_2} + \frac{\sigma_1}{\gamma_1}\tilde{\Theta}_1 \hat{\Theta}_1 + \frac{p_1 + 1}{p + 1} e_2^{p+1} + \Delta_1. \quad (47)$$

Step i ($i = 2, \ldots, n - 1$): Choose the ith LFc V_i as

$$V_i = \frac{1}{P_i} e_i^{P_i} + \frac{1}{2\gamma_i}\tilde{\Theta}_i^2 + V_{i-1}, \quad \left(\gamma_i \in R^+\right). \quad (48)$$

From (37) and (48), we have

$$\dot{V}_i = \dot{V}_{i-1} + e_i^{P_i - 1}\bar{f}_i(Z_i) + e_i^{P_i - 1}\left(\xi_{i+1}^{p_i} - \alpha_i^{p_i}\right) + e_i^{P_i - 1}\alpha_i^{p_i} - \frac{P_i - 1}{P_i} e_i^{P_i} - (\psi_i + \lambda_{i2}\varepsilon_i) e_i^{p+1} - \frac{1}{\gamma_i}\tilde{\Theta}_i \dot{\hat{\Theta}}_i \quad (49)$$

where $\bar{f}_i(Z_i) = (p_{i-1} + 1)e_i^{p_i}/(p + 1) + (P_i - 1)e_i/P_i + F_i(\bar{x}_n, \xi_{i+1}^{p_i}) + (\psi_i + \lambda_{i2}\varepsilon_i)e_i^{p_i} - \dot{\alpha}_{i-1}$, $\lambda_{i2}, \varepsilon_i \in R^+$, ψ_i will be shown below.

According to Lemmas 3–5, one has

$$e_i^{P_i - 1}\bar{f}_i(Z_i) \leq \frac{|e_i|^{P_i - 1}\tilde{\Theta}_i}{4a_i ||\Omega_i(\mathcal{Z}_i)||^2} + \delta_i + \phi_i e_i^{p+1} + \frac{P_i - 1}{P_i} e_i^{P_i} + \frac{\epsilon_i^{P_i}}{P_i} \quad (50)$$

$$e_i^{P_i - 1}\left(\xi_{i+1}^{p_i} - \alpha_i^{p_i}\right) \leq \frac{p_i + 1}{p + 1} e_{i+1}^{p+1} + \psi_i e_i^{p+1} \quad (51)$$

where $\Psi_i = a_i + (\sqrt{1 + \hat{\Theta}_i^2})/(4a_i||\Omega_i(\mathcal{Z}_i)||^2)$, $\Xi_i = p_i(2 + 2^{p_i - 2})$, $\psi_i = [(P_i - 1)\Xi_i^{(1+p)/(P_i - 1)}]/(1 + p) + [(2 + p)(\Xi_i |\pi_i|^{p_i - 1})^{(1+p)/(2+p)}]/(1 + p)$, and $\phi_i = [(P_i - 1)\Psi_i/(p + 1)]^{(p+1)/(P_i - 1)}[p_i/(P_i - 1)\delta_i]^{p_i/(P_i - 1)}$. a_i and δ_i are positive design parameters.

Design α_i and $\dot{\hat{\Theta}}_i$ as follows:

$$\alpha_i \triangleq -\Lambda_i e_i \quad (52)$$

$$\Lambda_i = \left[\lambda_{i1} + \frac{\lambda_{i2} e_i^{2(P_i r_2 - p - 1)}}{\sqrt{e_i^{2(P_i r_2 - p - 1)} + \varepsilon_i^2}} + \phi_i\right]^{\frac{1}{p_i}} \quad (53)$$

$$\dot{\hat{\Theta}}_i = \frac{\gamma_i |e_i|^{P_i - 1}}{4a_i ||\Omega_i(\mathcal{Z}_i)||^2} - \sigma_i \hat{\Theta}_i, \left(\hat{\Theta}_i(0) > 0\right) \quad (54)$$

where $\lambda_{i1}, \sigma_i \in R^+$.

Substituting (50)–(54) to (49), one has

$$\dot{V}_i \leq -\sum_{j=1}^{i}\lambda_{j1} e_j^{p+1} - \sum_{j=1}^{i}\lambda_{j2} e_j^{P_j r_2} + \sum_{j=1}^{i}\frac{\sigma_j}{\gamma_j}\tilde{\Theta}_j \hat{\Theta}_j + \frac{p_i + 1}{p + 1} e_{i+1}^{p+1} + \sum_{j=1}^{i}\Delta_j \quad (55)$$

where $\Delta_j = \delta_j + \epsilon_j^{P_j}/P_j$.

Step n: Choose the nth LFc V_n as follows:

$$V_n = \frac{1}{P_n} e_n^{P_n} + \frac{1}{2\gamma_n}\tilde{\Theta}_n^2 + V_{n-1}, \quad \gamma_n \in R^+. \quad (56)$$

Then, one has

$$\dot{V}_n = \dot{V}_{n-1} + e_n^{P_n - 1}\bar{f}_n(Z_n) - \frac{(P_n - 1)e_n^{P_n}}{P_n} - \lambda_{n1}\varepsilon_n e_n^{p+1} + e_n^{P_n - 1}\mu_n g_n(\bar{x}_n)u^{p_n} - \frac{\tilde{\Theta}_n \dot{\hat{\Theta}}_n}{\gamma_n} \quad (57)$$

where $\bar{f}_n(Z_n) = F_n(\bar{x}_n) + (P_n - 1)e_n/P_n + (p_{n-1} + 1)e_n^{p_n}/(p + 1) - \dot{\alpha}_{n-1} + \lambda_{n1}\varepsilon_n e_n^{p_n}$, $\lambda_{n1}, \varepsilon_n \in R^+$.

Similar to (40) and (50), one has

$$e_n^{P_n - 1}\bar{f}_n(Z_n) \leq \frac{|e_n|^{P_n - 1}\tilde{\Theta}_n}{4a_n ||\Omega_n(\mathcal{Z}_n)||^2} + \delta_n + \phi_n e_n^{p+1} + \frac{P_n - 1}{P_n} e_n^{P_n} + \frac{\epsilon_n^{P_n}}{P_n} \quad (58)$$

where $\Psi_n = a_n + (\sqrt{1 + \hat{\Theta}_n^2})/(4a_n||\Omega_n(\mathcal{Z}_n)||^2)$, $\phi_n = [(P_n - 1)\Psi_n/(p + 1)]^{(p+1)/(P_n - 1)}[p_n/(P_n - 1)\delta_n]^{p_n/(P_n - 1)}$. a_n and δ_n are positive design parameters.

Design u_n and $\dot{\hat{\Theta}}_n$ as follows:

$$u \triangleq -\Lambda_n e_n \quad (59)$$

$$\Lambda_n = \left[\frac{1}{\mu_n \underline{g}}\left(\lambda_{n1} + \frac{\lambda_{n2} e_n^{2(P_n r_2 - p - 1)}}{\sqrt{e_n^{2(P_n r_2 - p - 1)} + \varepsilon_n^2}} + \phi_n\right)\right]^{\frac{1}{p_n}} \quad (60)$$

$$\dot{\hat{\Theta}}_n = \frac{\gamma_n |e_n|^{P_n - 1}}{4a_n ||\Omega_n(\mathcal{Z}_n)||^2} - \sigma_n \hat{\Theta}_n, \left(\hat{\Theta}_n(0) > 0\right) \quad (61)$$

where $\lambda_{n1}, \sigma_n \in R^+$.

According to (59)–(61), we have

$$\dot{V}_n \leq -\sum_{j=1}^{n}\left[\lambda_{j1} e_j^{p+1} + \lambda_{j2} e_j^{P_j r_2} - \frac{\sigma_j \tilde{\Theta}_j \hat{\Theta}_j}{\gamma_j}\right] + \Delta_0 \quad (62)$$

where $\Delta_0 = \sum_{j=1}^{n}\Delta_j$.

One can obtain from Lemma 5 that $e_i^{P_i} \leq \nu_i + e_i^{p+1}\pi_i$, where $\pi_i = (P_i/(p+1))^{(p+1)/P_i}((p_i - 1)/P_i\nu_i)^{(p_i - 1)/P_i}$, $\nu_i \in R^+$. Thus, $-e_i^{p+1} \leq (-e_i^{P_i} + \nu_i)/\pi_i$, $i = 1, \ldots, n$. Then, we have

$$\dot{V}_n \leq -\sum_{j=1}^{n}\left[\frac{\lambda_{j1} e_j^{P_j}}{\pi_j} + \lambda_{j2} e_j^{P_j r_2} - \frac{\sigma_j \tilde{\Theta}_j \hat{\Theta}_j}{\gamma_j}\right] + \bar{\Delta}_0 \quad (63)$$

where $\bar{\Delta}_0 = \Delta_0 + \sum_{j=1}^{n}(\lambda_{j1}\nu_j/\pi_j)$.

Since $\tilde{\Theta}_j \hat{\Theta}_j \leq -\tilde{\Theta}_j^2/2 + \Theta_j^2/2$, then one has

$$\dot{V}_n \leq -\sum_{j=1}^{n}\left[\frac{\lambda_{j1} e_j^{P_j}}{\pi_j} + \lambda_{j2} e_j^{P_j r_2} + \frac{\sigma_j \tilde{\Theta}_j^2}{2\gamma_j} - \frac{\sigma_j \Theta_j^2}{2\gamma_j}\right] + \bar{\Delta}_0. \quad (64)$$

On the basis of Lemmas 3 and 4, we have

$$\left(\sum_{j=1}^{n}\frac{\lambda_{j1} e_j^{P_j}}{\pi_j}\right)^{r_1} \leq \sum_{j=1}^{n}\frac{\lambda_{j1} e_j^{P_j}}{\pi_i} + (1 - r_1) r_1^{\frac{r_1}{1 - r_1}} \quad (65)$$

$$-\sum_{j=1}^{n}\lambda_{j2}e_j^{P_j r_2}\leq-\frac{\bar{\lambda}_2\left(\sum_{j=1}^{n}e_j^{P_j}\right)^{r_2}}{n^{r_2-1}} \tag{66}$$

$$\left(\sum_{j=1}^{n}\frac{\sigma_j\tilde{\Theta}_j^2}{4\lambda_j}\right)^{r_1}\leq\sum_{j=1}^{n}\frac{\sigma_j\tilde{\Theta}_j^2}{4\lambda_j}+(1-r_1)r_1^{\frac{r_1}{1-r_1}} \tag{67}$$

$$\left(\sum_{j=1}^{n}\frac{\sigma_j\tilde{\Theta}_j^2}{4\lambda_j}\right)^{r_2}\leq\sum_{j=1}^{n}\frac{\sigma_j\tilde{\Theta}_j^2}{4\lambda_j}+(1-r_2)r_2^{\frac{r_2}{1-r_2}} \tag{68}$$

where $r_1\in(0,1)$, $\bar{\lambda}_2=\min_{1\leq j\leq n}\{\lambda_{j2}\}$.

According to (65)–(68), one can obtain

$$\begin{aligned}\dot{V}_n\leq&-\left(\sum_{j=1}^{n}\frac{\lambda_{j1}}{\pi_j}e_j^{P_j}\right)^{r_1}-\frac{\bar{\lambda}_2}{n^{r_2-1}}\left(\sum_{j=1}^{n}e_j^{P_j}\right)^{r_2}\\&-\left(\sum_{j=1}^{n}\frac{\sigma_j\tilde{\Theta}_j^2}{4\lambda_j}\right)^{r_1}-\left(\sum_{j=1}^{n}\frac{\sigma_j\tilde{\Theta}_j^2}{4\lambda_j}\right)^{r_2}+\Delta\end{aligned} \tag{69}$$

where $\Delta=\bar{\Delta}_0+\sum_{j=1}^{n}(\sigma_j\Theta_j^2)/(2\gamma_j)+2(1-r_1)r_1^{r_1/(1-r_1)}$.

Let $\hat{\lambda}_2=\min_{1\leq j\leq n}\{(\bar{\lambda}_2P_j^{r_2})/n^{r_2-1},(\sigma_j/2)^{r_2}/n^{r_2-1}\}$, $\hat{\lambda}_1=\min_{1\leq j\leq n}\{(\lambda_{j1}P_j/\pi_j)^{r_1},(\sigma_j/2)^{r_1}\}$. Then, (69) can be rewritten as follows:

$$\begin{aligned}\dot{V}_n\leq&-\hat{\lambda}_1\left[\left(\sum_{j=1}^{n}\frac{e_j^{P_j}}{P_j}\right)^{r_1}+\left(\sum_{j=1}^{n}\frac{\tilde{\Theta}_j^2}{2\lambda_j}\right)^{r_1}\right]\\&-\hat{\lambda}_2\left[\left(\sum_{j=1}^{n}\frac{e_j^{P_j}}{P_j}\right)^{r_2}+\left(\sum_{j=1}^{n}\frac{\tilde{\Theta}_j^2}{2\lambda_j}\right)^{r_2}\right]+\Delta.\end{aligned} \tag{70}$$

According to Lemma 4, we have

$$V_n^{r_1}\leq\left(\sum_{j=1}^{n}\frac{e_j^{P_j}}{P_j}\right)^{r_1}+\left(\sum_{j=1}^{n}\frac{\tilde{\Theta}_j^2}{2\lambda_j}\right)^{r_1} \tag{71}$$

$$V_n^{r_2}\leq 2^{r_2-1}\left[\left(\sum_{j=1}^{n}\frac{e_j^{P_j}}{P_j}\right)^{r_2}+\left(\sum_{j=1}^{n}\frac{\tilde{\Theta}_j^2}{2\lambda_j}\right)^{r_2}\right]. \tag{72}$$

Based on (70)–(72), we have

$$\dot{V}_n\leq-\lambda_1V_n^{r_1}-\lambda_2V_n^{r_2}+\Delta \tag{73}$$

where $\lambda_1=\hat{\lambda}_1$ and $\lambda_2=\hat{\lambda}_2/2^{r_2-1}$.

The flow chart of control algorithm is shown in Fig. 1.

Remark 9: It is worth noting that the use of FLSs to approximate nonlinear functions must be in a compact set. Therefore, when using the proposed method to deal with unconstrained HONSs, it is need to replace ξ_i, μ_i, v_i, and α_0 in the case of state constraint by x_i, 1, 0, and y_d, respectively, and redesigning the controller and altering the design parameters are not needed. Of course, the use of FLSs to approximate unknown nonlinear functions in control design is not the first time proposed in this article, and similar methods also have been proposed in [9], [10], and [11] and their references.

Remark 10: The singularity problem is a common problem in FTC schemes. The early fixed-time dynamic surface control method proposed in [14], [15], and [42] is an effective way

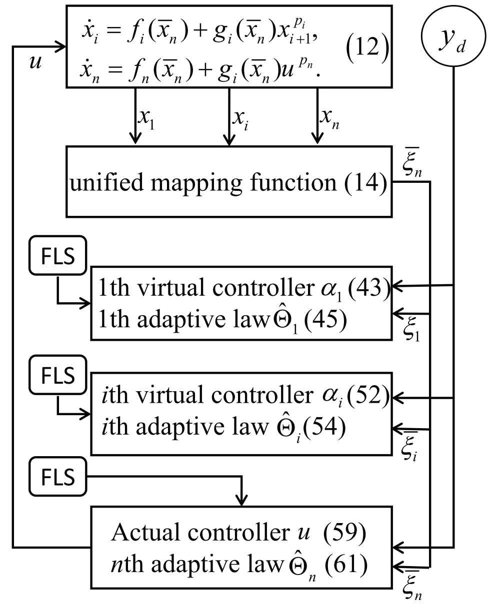

Fig. 1. Flow chart of control algorithm.

to solve this problem. In this article, we take the virtual control function as a part of the unknown nonlinear function, and directly use the FLS to approximate it so as to avoid the singularity caused by the repeated derivation of the virtual control function, and the similar method is also presented in [47]. It is noted that the virtual controller designed in [33] and [47] is continuous but not derivable (i.e., when $e_i=0$, the derivative of α_i does not exist), so the methods in [33] and [47] cannot completely avoid singularity. It can be known from (43), (44), (52), and (53) that the proposed method effectively avoids this problem.

C. Stability Analysis

Theorem: Consider the HONSs (12) with Assumptions 1 and 2, under the initial condition $x_i(0)\in\mathcal{A}_i$, the designed adaptive control algorithm ensures the following.

1) All of the closed-loop signals are bounded, and the full-state constraints are not violated under removal FCs.
2) The tracking error $z_1=y-y_d$ converges to a zero-centered neighborhood within a fixed time.

Proof: 1) Select the entire LF $V=V_n$, from (73), one has

$$\dot{V}\leq-\lambda_1V^{r_1}-\lambda_2V^{r_2}+\Delta. \tag{74}$$

According to Lemma 1, one has

$$V\leq\min\left\{\left(\frac{\Delta}{(1-\eta)\lambda_1}\right)^{\frac{1}{r_1}},\left(\frac{\Delta}{(1-\eta)\lambda_2}\right)^{\frac{1}{r_2}}\right\} \tag{75}$$

with

$$T\leq\frac{1}{\lambda_1\eta(1-r_1)}+\frac{1}{\lambda_2\eta(r_2-1)} \tag{76}$$

where $\eta\in(0,1)$.

According to the expression of V, one has

$$|e_i| \le \min\left\{P_i\left[\left(\frac{\Delta}{(1-\eta)\lambda_1}\right)^{\frac{1}{r_1}}\right]^{\frac{1}{P_i}} P_i\left[\left(\frac{\Delta}{(1-\eta)\lambda_2}\right)^{\frac{1}{r_2}}\right]^{\frac{1}{P_i}}\right\} \tag{77}$$

and

$$\left|\tilde{\Theta}_i\right| \le \min\left\{\sqrt{2\gamma_i\left(\frac{\Delta}{(1-\eta)\lambda_1}\right)^{\frac{1}{r_1}}}\sqrt{2\gamma_i\left(\frac{\Delta}{(1-\eta)\lambda_2}\right)^{\frac{1}{r_2}}}\right\}. \tag{78}$$

It means that $\tilde{\Theta}_i$ and e_i are bounded. It can be known from the expressions of α_i and u that α_i and u are bounded, so is ξ_i. From Lemma 6, one has $x_i \in \mathcal{A}_i$. Sum up, all of the closed-loop signals are bounded, and the full-state constraints are not violated. Based on the analysis of Remark 4, we know that the FCs are eliminated.

2) Based on (77), we know that e_1 converges to a zero-centered neighborhood within a fixed time by choosing the suitable parameter. From (14) and (36), one has $e_1 = \mathcal{Q}z_1$, where

$$\mathcal{Q} = \frac{1}{2}\left[\frac{H_1^2 - \underline{H}_1}{(H_1 - x_1)(H_1 - y_d)} + \frac{L_1^2 + \bar{L}_1}{(y_d - L_1)(x_1 - L_1)}\right].$$

Since $\mathcal{Q}$ is bounded, the tracking error z_1 can converge to a zero-centered neighborhood within a fixed time by adjusting appropriate parameters. Then, conclusion 2) holds. ■

Remark 11: Note that the proposed controller and adaptive law involve some design parameters. On the basis of the bounded analysis of the closed-loop system, the convergence time can be reduced by appropriately increasing the values of λ_{i1} and σ_i, and the tracking error can be reduced by appropriately increasing the values of λ_{i2} and γ_i. Of course, the actual simulation also needs to debug the parameters according to the actual needs, and so far, the adjustment of some parameters can only be guided by experience.

IV. Simulation

Example 1: Give the following HONS:

$$\begin{cases} \dot{x}_1 = \sin x_1 e^{-0.5x_2} + \left(1 + x_1^2\right)x_2^3 \\ \dot{x}_2 = x_1 x_2^2 + (3 - \cos x_1 \cos x_2)u^3 \\ y = x_1. \end{cases} \tag{79}$$

Let $y_d = 0.5\sin t$, $L_1 = -0.2 \times 2^{-0.3t} - 0.5\sin t - 0.75$, $H_1 = 0.3\times 2^{-0.3t} + 0.5\sin t + 0.5$, $L_2 = -1 - 0.2\sin(t+5)$, $H_2 = 1.5 - 2^{-0.3t}$, $\bar{L}_i = 0.1$, and $\underline{H}_i = -0.1$. Let $\lambda_{1j} = 15$, $\lambda_{2j} = 5$, $\sigma_i = 1$, $a_i = 0.1 (i, j = 1, 2)$, $\gamma_1 = 10$, $\gamma_2 = 30$, $r_2 = 3$, and $[x_1(0), x_2(0), \hat{\Theta}_1(0), \hat{\Theta}_2(0)]^T = [0.1, -0.1, 3, 4]^T$.

In order to verify the effectiveness of our method, we compare our method with the existing advanced methods. For the case where the upper and lower constraint boundaries are positive and negative functions (i.e., case ①), Figs. 2 and 3 show the comparison between the proposed method and the method in [31]. It can be seen from Fig. 2 that under the premise of not violating the state constraint, the system output can better track the desired signal under the two methods. Fig. 3 shows the tracking error curves of the two methods, which means that when the design parameters are the same, the tracking error of the proposed method is smaller than that of the method in [31]. Moreover, the method proposed in [31] can not predict the settling time in advance, but the proposed method can. Fig. 4 shows the curves of x_2 and α_1, obviously, the state constraint is not violated, that is, $L_2 < x_2 < H_2$, and α_1 is not within the constraint range, which verifies that the proposed method effectively removes the FC existing in methods proposed in [24], [25], [26], [27], [28], and [30] (see Remark 4).

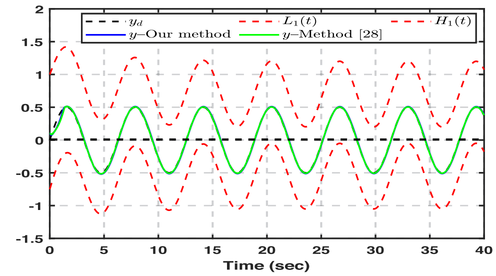

Fig. 2. Tracking curves under our method and the method in [31].

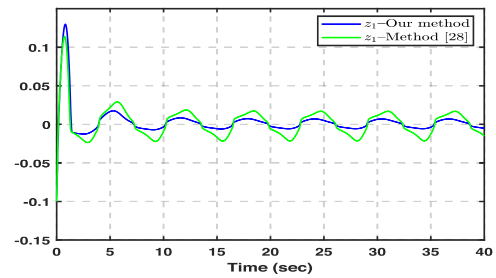

Fig. 3. Tracking error curves under our method and the method in [31].

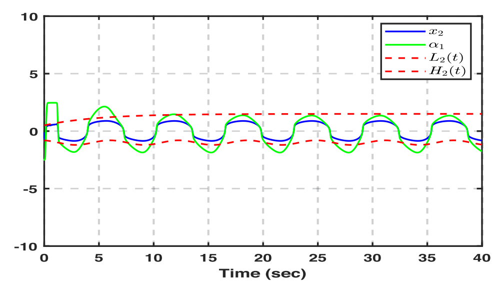

Fig. 4. Curves of x_2 and α_1.

In addition, we also compare our approach with that of [19], which also applies to cases ①–⑥. Let $y_d = 1.5\sin(0.5t)$, $L_1 = 1.5\sin(0.5t) - 0.5 \times 2^{-0.5t} - 0.8$, $H_1 = 0.1 \times 2^{-0.3t} + 1.5\sin(0.5t) + 1.2$, $L_2 = -1 - 0.2\sin(t+5)$, and $H_2 = 1.5 - 2^{-0.3t}$. Obviously, L_1 and H_1 are neither strictly positive nor strictly negative. Figs. 5 and 6 show the tracking curves and tracking error curves of the two methods under the same design parameters. When the design parameters are the same, the tracking error of the proposed method is smaller than that of [19]. Note that the method in [19] is limited to general feedback nonlinear systems, and the settling time of the system cannot be predicted in advance.

To verify that the proposed method is applicable to HONSs with or without multitype state constraints, Fig. 7 gives the

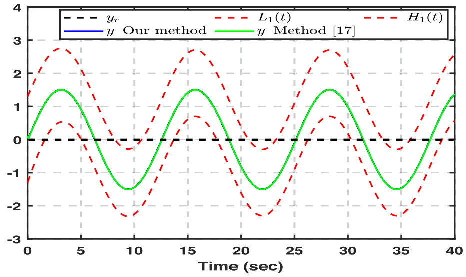

Fig. 5. Tracking curves of our method and the method in [19].

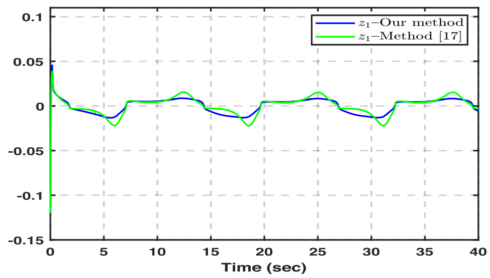

Fig. 6. Tracking error curves of our method and the method in [19].

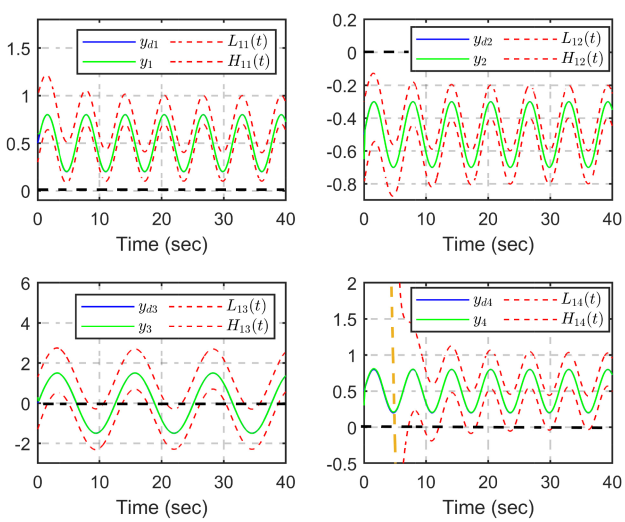

Fig. 7. Results under different constraint types.

simulation results under multitype state constraints. In Fig. 7, subgraphs (a) and (b) show the tracking curves for the case where both the upper and lower constraint boundaries are strictly positive and strictly negative functions, respectively. Subgraph (c) shows the tracking curve for the case where the upper and lower constraint boundaries are neither strictly positive nor strictly negative functions. Furthermore, select $L_1 = -\infty, H_1 = +\infty$ when $t \leq 5$, $L_1 = 0.2 - 0.2 \times 2^{-0.3t} - 0.3\sin t - 1/(t-5)$, $H_1 = 0.3 \times 2^{-0.3t} + 1/(t-5) + 0.3\sin t + 0.8$ when $t > 5$, Subfigure (d) shows the result of acting interactively under constraints and unconstraints. It is noted that when $L_1 = -\infty, H_1 = +\infty$, it is need to replace ξ_i, μ_i, v_i, and α_0 in the case of state constraint by x_i, 1, 0, and y_d, respectively, and redesigning the controller and altering the design parameters are not needed.

Remark 12: It is worth noting that when the constraint boundaries are neither strictly positive nor strictly negative, it can be seen from Remarks 4 and 5 that none of the existing methods are suitable for the above constraint types except for [19], [20]. However, the methods in [19] and [20] are not suitable for HONSs without state constraints. From this point of view, the proposed approach has a wider range of application. In addition, from the control performance, the proposed method can achieve better tracking accuracy under the same design parameters, and the proposed method can predict the settling time in advance, but the methods in [19], [20], and [31] can not.

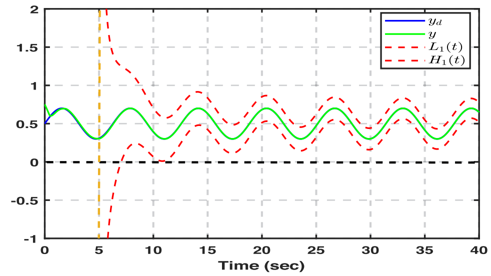

Fig. 8. Desired signal y_d, system output y and its boundary functions.

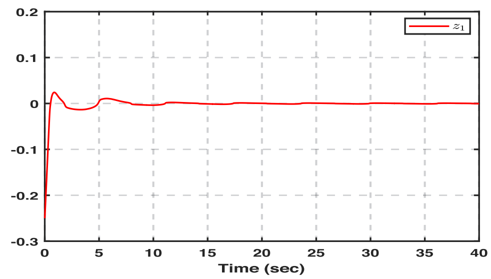

Fig. 9. Tracking error z_1.

Example 2: Consider a reduced-order model of boiler-turbine unit [31] as follows:

$$\begin{cases} \dot{x}_1 = x_2^3 \\ \dot{x}_2 = \frac{x_1^2}{1+x_2^2} + u \\ y = x_1 \end{cases} \tag{80}$$

where x_1 and x_2 denote the drum and reheater pressures, respectively, and u denotes the position of the control valve. Let $y_d = 0.5 + 0.2\sin t$, $L_1 = -\infty$, and $H_1 = +\infty$, when $t \leq 5$,$L_1 = 0.4 - 0.2 \times 2^{-0.3t} + 0.2\sin t - 1/(t-5)$, $H_1 = 0.6 + 0.1 \times 2^{-0.3t} + 0.2\sin t + 1/(t-5)$ when $t > 5$, $L_2 = -1 - 0.2\sin(t+5)$, $H_2 = 1.5 - 2^{-0.3t}$, $\bar{L}_i = 0.1$, and $\underline{H}_i = -0.1$. Let $\lambda_{1j} = 15, \lambda_{2j} = 5, \sigma_i = 1$, $a_i = 0.1 (i, j = 1, 2)$, $\gamma_1 = 10, \gamma_2 = 30$, $r_2 = 3$, and $[x_1(0), x_2(0), \hat{\Theta}_1(0), \hat{\Theta}_2(0)]^T = [0.75, -0.1, 3, 4]^T$. The results are shown in Figs. 8–12. Figs. 8 and 9 show the tracking curves and tracking error curves under the alternating of constraint and nonconstraint, respectively, which implies that the proposed method can achieve good tracking without violating state constraints. Fig. 10 shows the curve of x_2 and α_1, which implies that the proposed method can remove the FC. Figs. 11 and 12 show the curves of the control input u and the adaptive parameters $\hat{\Theta}_1$ and $\hat{\Theta}_2$, respectively.

V. Conclusion

In this article, a novel fuzzy control algorithm for HONSs subject to multitype state constraints is presented. The difference from the conventional approaches is that the proposed

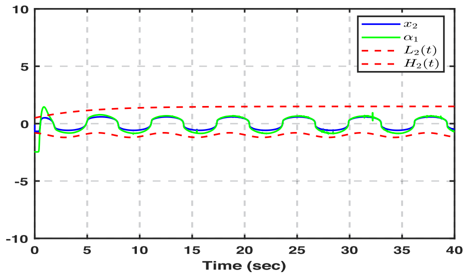

Fig. 10. Curves of x_2 and α_1.

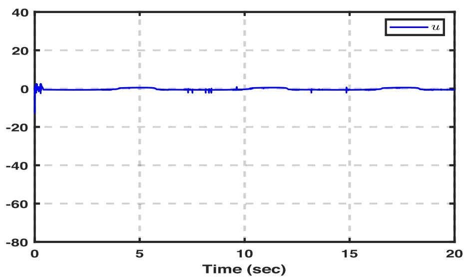

Fig. 11. Trajectory of u.

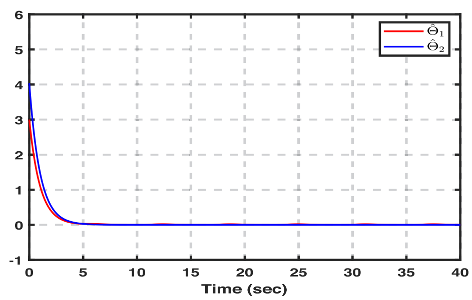

Fig. 12. Trajectories of $\hat{\Theta}_1$ and $\hat{\Theta}_2$.

scheme effectively eliminates the FCs, and capable of accommodating multiple type time-varying state constraints (cases ①–⑥). In addition, a technical design makes the proposed method also suitable for unconstrained HONSs. Combined with FLS and FTST, the proposed algorithm ensure the fixed-time stability. Of course, the FTC algorithm proposed in this article still has the limitation of many design parameters. Recently, some prescribed time control schemes [48], [49], [50] were proposed successively. In the future, we will focus on prescribed time control algorithm for HONSs with multitype state constraints.

References

[1] A. Bemporad, "Reference governor for constrained nonlinear systems," *IEEE Trans. Autom. Control*, vol. 43, no. 3, pp. 415–419, Mar. 1998.

[2] E. G. Gilbert and I. Kolmanovsky, "Nonlinear tracking control in the presence of state and control constraints: A generalized reference governor," *Automatica*, vol. 38, no. 12, pp. 2063–2073, Dec. 2002.

[3] D. Q. Mayne, J. B. Rawlings, C. V. Rao, and P. O. M. Scokaert, "Constrained model predictive control: Stability and optimality," *Automatica*, vol. 36, no. 6, pp. 789–814, Jun. 2000.

[4] T. Hu and Z. Lin, *Control Systems With Actuator Saturation: Analysis and Design*. Boston, MA, USA: Birkhäuser, 2001.

[5] K. B. Ngo, R. Mahony, and Z. P. Jiang, "Integrator backstepping using barrier functions for systems with multiple state constraints," in *Proc. 44th IEEE Conf. Decis. Control*, 2005, pp. 8306–8312.

[6] Y.-J. Liu and S. C. Tong, "Barrier Lyapunov functions for Nussbaum gain adaptive control of full state constrained nonlinear systems," *Automatica*, vol. 76, pp. 143–152, Feb. 2017.

[7] L. B. Wu, J. H. Park, X. P. Xie, and N. N. Zhao, "Adaptive fuzzy tracking control for a class of uncertain switched nonlinear systems with full-state constraints and input saturations," *IEEE Trans. Cybern.*, vol. 51, no. 12, pp. 6054–6065, Dec. 2021.

[8] X. H. Li, X. Li, and S. C. Tong, "Backstepping-based fuzzy adaptive stabilization of reaction-diffusion equation with state constraints," *IEEE Trans. Cybern.*, early access, Dec. 19, 2022, doi: 10.1109/TCYB.2022.3227899.

[9] L. Liu, A. Chen, and Y.-J. Liu, "Adaptive fuzzy output-feedback control for switched uncertain nonlinear systems with full-state constraints," *IEEE Trans. Cybern.*, vol. 52, no. 8, pp. 7340–7351, Aug. 2022.

[10] L. Tang, M. Yang, Y.-J. Liu, and S. C. Tong, "Adaptive output feedback fuzzy fault-tolerant control for nonlinear full-state-constrained switched systems," *IEEE Trans. Cybern.*, vol. 53, no. 4, pp. 2325–2334, Apr. 2023, doi: 10.1109/TCYB.2021.3116950.

[11] Y. H. Liu, Y. Liu, Y. F. Liu, and C. Y. Su, "Adaptive fuzzy control with global stability guarantees for unknown strict-feedback systems using novel integral barrier Lyapunov functions," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 52, no. 7, pp. 4336–4348, Jul. 2022.

[12] K. Zhao and Y. Song, "Removing the feasibility conditions imposed on tracking control designs for state-constrained strict-feedback systems," *IEEE Trans. Autom. Control*, vol. 64, no. 3, pp. 1265–1272, Mar. 2019.

[13] K. Zhao and Y. Song, "Neuroadaptive robotic control under time-varying asymmetric motion constraints: A feasibility-condition-free approach," *IEEE Trans. Cybern.*, vol. 50, no. 1, pp. 15–24, Jan. 2020.

[14] Y. G. Yao, J. Q. Tan, J. Wu, and X. Zhang, "Event-triggered fixed-time adaptive fuzzy control for state-constrained stochastic nonlinear systems without feasibility conditions," *Nonlinear Dyn.*, vol. 105, pp. 403–416, Jun. 2021.

[15] Y. G. Yao, J. Q. Tan, J. Wu, and X. Zhang, "Decentralized fixed-time control for state-constrained stochastic systems via nonlinear state-dependent function approach," *Int. J. Robust Nonlinear Control*, vol. 32, no. 8, pp. 4923–4945, 2022.

[16] Z. G. Feng, R.-B. Li, and X. J. Jing, "Neuroadaptive control for active suspension systems with time-varying motion constraints: A feasibility-condition-free method," *IEEE Trans. Cybern.*, early access, Nov. 8, 2022, doi: 10.1109/TCYB.2022.3217404.

[17] K. Zhao, Y. D. Song, and Z. R. Zhang, "Tracking control of MIMO nonlinear systems under full state constraints: A single-parameter adaptation approach free from feasibility conditions," *Automatica*, vol. 107, pp. 52–60, Sep. 2019.

[18] Y. Cao, C. Wen, and Y. Song, "A unified event-triggered control approach for uncertain pure-feedback systems with or without state constraints," *IEEE Trans. Cybern.*, vol. 51, no. 3, pp. 1262–1271, Mar. 2021.

[19] K. Zhao, Y. D. Song, C. L. P. Chen, and L. Chen, "Control of nonlinear systems under dynamic constraints: A unified barrier function-based approach," *Automatica*, vol. 119, Sep. 2020, Art. no. 109102.

[20] K. Zhao, L. Chen, W. Meng, and L. Zhao, "Unified mapping function-based neuroadaptive control of constrained uncertain robotic systems," *IEEE Trans. Cybern.*, early access, Jan. 4, 2022, doi: 10.1109/TCYB.2021.3135893.

[21] C.-C. Chen and Z.-Y. Sun, "A unified approach to finite-time stabilization of high-order nonlinear systems with an asymmetric output constraint," *Automatica*, vol. 111, Jan. 2020, Art. no. 108581.

[22] L. Fang, S. Ding, J. H. Park, and L. Ma, "Adaptive fuzzy control for stochastic high-order nonlinear systems with output constraints," *IEEE Trans. Fuzzy Syst.*, vol. 29, no. 9, pp. 2635–2646, Sep. 2021.

[23] L. Fang, S. Ding, J. H. Park, and L. Ma, "Adaptive fuzzy control for non-triangular stochastic high-order nonlinear systems subject to asymmetric output constraints," *IEEE Trans. Cybern.*, vol. 52, no. 2, pp. 1280–1291, Feb. 2022, doi: 10.1109/TCYB.2020.3000920.

[24] W. Sun, S. F. Su, G. Dong, and W. Bai, "Reduced adaptive fuzzy tracking control for high-order stochastic nonstrict feedback nonlinear system with full-state constraints," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 51, no. 3, pp. 1496–1506, Mar. 2021.

[25] W. Sun, S. F. Su, Y. Wu, J. Xia, and V. T. Nguyen, "Adaptive fuzzy control with high-order barrier Lyapunov functions for high-order uncertain nonlinear systems with full-state constraints," *IEEE Trans. Cybern.*, vol. 50, no. 8, pp. 3424–3432, Aug. 2020.

[26] N. Wang, Y. Wang, G. Wen, M. Lv, and F. Zhang, "Fuzzy adaptive constrained consensus tracking of high-order multi-agent networks: A new event-triggered mechanism," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 52, no. 9, pp. 5468–5480, Sep. 2022.

[27] Y. Wu and X.-J. Xie, "Adaptive fuzzy control for high-order nonlinear time-delay systems with full-state constraints and input saturation," *IEEE Trans. Fuzzy Syst.*, vol. 28, no. 8, pp. 1652–1663, Aug. 2020.

[28] N. Wang, F. Tao, Z. Fu, and S. Song, "Adaptive fuzzy control for a class of stochastic strict feedback high-order nonlinear systems with full-state constraints," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 52, no. 1, pp. 205–213, Jan. 2022.

[29] Q. Zhang and D. He, "Adaptive fuzzy sliding exact tracking control based on high-order log-type time-varying BLFs for high-order nonlinear systems," *IEEE Trans. Fuzzy Syst.*, vol. 31, no. 1, pp. 14–24, Jan. 2023.

[30] C. Guo, R. Xie, and X.-J. Xie, "Adaptive control of full-state constrained high-order nonlinear systems with time-varying powers," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 51, no. 8, pp. 5189–5197, Aug. 2021.

[31] X.-J. Xie, Y. Wu, and Z.-G. Hou, "Further results on adaptive practical tracking for high-order nonlinear systems with full-state constraints," *IEEE Trans. Cybern.*, vol. 52, no. 10, pp. 9978–9985, Oct. 2022, doi: 10.1109/TCYB.2021.3069865.

[32] Y. Wu, X.-J. Xie, and Z.-G. Hou, "Adaptive fuzzy asymptotic tracking control of state-constrained high-order nonlinear time-delay systems and its applications," *IEEE Trans. Cybern.*, vol. 52, no. 3, pp. 1671–1680, Mar. 2022.

[33] Y. G. Yao, J. Q. Tan, J. Wu, and X. Zhang, "A unified fuzzy control approach for stochastic high-order nonlinear systems with or without state constraints," *IEEE Trans. Fuzzy Syst.*, vol. 30, no. 10, pp. 4530–4540, Oct. 2022.

[34] S. P. Bhat and D. S. Bernstein, "Continuous finite-time stabilization of the translational and rotational double integrators," *IEEE Trans. Autom. Control*, vol. 43, no. 5, pp. 678–682, May 1998.

[35] S. Li, L. Ding, H. B. Gao, Y. J. Liu, L. Huang, and Z. Q. Deng, "Adaptive fuzzy finite-time tracking control for nonstrict full states constrained nonlinear system with coupled dead-zone input," *IEEE Trans. Cybern.*, vol. 52, no. 2, pp. 1138–1149, Feb. 2022.

[36] H. F. Min, S. Y. Xu, and Z. Q. Zhang, "Adaptive finite-time stabilization of stochastic nonlinear systems subject to full-state constraints and input saturation," *IEEE Trans. Autom. Control*, vol. 66, no. 3, pp. 1306–1313, Mar. 2021.

[37] Z. Y. Sun, Y. R. Peng, C. Y. Wen, and C.-C. Chen, "Fast finite-time adaptive stabilization of high-order uncertain nonlinear system with an asymmetric output constraint," *Automatica*, vol. 121, Nov. 2020, Art. no. 109170.

[38] L. Fang, L. Ma, S. Ding, and D. Zhao, "Finite-time stabilization for a class of high-order stochastic nonlinear systems with an output constraint," *Appl. Math. Comput.*, vol. 358, pp. 63–79, Oct. 2019.

[39] A. Polyakov, D. Efimov, and W. Perruquetti, "Finite-time and fixed-time stabilization: Implicit Lyapunov function approach," *Automatica*, vol. 51, pp. 332–340, Jan. 2015.

[40] X. Jin, "Adaptive fixed-time control for MIMO nonlinear systems with asymmetric output constraints using universal barrier functions," *IEEE Trans. Autom. Control*, vol. 64, no. 7, pp. 3046–3053, Jul. 2019.

[41] M. Chen, H. Q. Wang, and X. P. Liu, "Adaptive practical fixed-time tracking control with prescribed boundary constraints," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 68, no. 4, pp. 1716–1726, Apr. 2021.

[42] Y. G. Yao, J. Q. Tan, J. Wu, and X. Zhang, "Event-triggered fixed-time adaptive neural dynamic surface control for stochastic non-triangular structure nonlinear systems," *Inf. Sci.*, vol. 569, pp. 527–543, Aug. 2021.

[43] Y. G. Yao, J. Q. Tan, J. Wu, and X. Zhang, "Event-triggered fixed-time adaptive neural tracking control for stochastic non-triangular structure nonlinear systems," *Neural Comput. Appl.*, vol. 33, pp. 15887–15899, Sep. 2021.

[44] S. Sui, C. L. P. Chen, and S. Tong, "A novel full errors fixed-time control for constraint nonlinear systems," *IEEE Trans. Autom. Control*, vol. 68, no. 4, pp. 2568–2575, Apr. 2023, doi: 10.1109/TAC.2022.3200962.

[45] H. Li, C. Hua, and K. Li, "Fixed-time stabilization for interconnected high-order nonlinear systems with dead-zone input and output constraint," *J. Franklin Inst.*, vol. 358, no. 14, pp. 6923–6940, Sep. 2021.

[46] Z. Y. Sun, C. Zhou, C. Y. Wen, and C.-C. Chen, "Adaptive event-triggered fast finite-time stabilization of high-order uncertain nonlinear systems and its application in maglev systems," *IEEE Trans. Cybern.*, early access, Nov. 29, 2022, doi: 10.1109/TCYB.2022.3220742.

[47] J. Ma, H. Wang, and J. Qiao, "Adaptive neural fixed-time tracking control for high-order nonlinear systems," *IEEE Trans. Neural Netw. Learn. Syst.*, early access, Jun. 6, 2022, doi: 10.1109/TNNLS.2022.3176625.

[48] S. Sui, C. L. P. Chen, and S. Tong, "Finite-time adaptive fuzzy prescribed performance control for high-order stochastic nonlinear systems," *IEEE Trans. Fuzzy Syst.*, vol. 30, no. 7, pp. 2227–2240, Jul. 2022.

[49] Y. Cao, J. F. Cao, and Y. D. Song, "Practical prescribed time tracking control over infinite time interval involving mismatched uncertainties and non-vanishing disturbances," *Automatica*, vol. 136, Feb. 2022, Art. no. 110050.

[50] Y. Cao, J. F. Cao, and Y. D. Song, "Practical prescribed time control of Euler–Lagrange systems with partial/full state constraints: A settling time regulator-based approach," *IEEE Trans. Cybern.*, vol. 52, no. 12, pp. 13096–13105, Dec. 2022, doi: 10.1109/TCYB.2021.3100764.

[51] C.J. Qian and W. Lin, "Practical output tracking of nonlinear systems with uncontrollable unstable linearization," *IEEE Trans. Autom. Control*, vol. 47, no. 1, pp. 21–36, Jan. 2002.

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